Foreign Exchange Reserves–Towards a Sustainable Allocation

CHARITH GAMAGE*

Abstract

In recent times, an increasing volatility in the foreign reserves of small open economies, largely driven by the challenging global environment has been observed. These economies are mostly running a current account deficit and are facing a major challenge of balancing external sector stability while generating a reasonable return out of reserves. This paper focuses on a top-down approach to allocating reserve assets in a way where maintaining a quantifiable buffer for external sector stability and assures a prudent a return generation framework via the excess foreign exchange reserves. The amount of excess reserves is derived from the concept of the "cost" of holding reserves. The writer's reserve asset allocation framework, for the return generation objective, expands beyond the classical mean-variance techniques for risk-return tradeoffs. Thereby, this paper explores a prudent approach; extracting risk factors out of empirical distributions of a typical reserve' investments and using Bayesian techniques to model a sustainable reserve asset allocation. Findings suggest that outlining a prudent reserve management framework, suiting the country's external sector dynamics stands as the cornerstone for sustainability. As a result, expected returns and asset allocations could be significantly changed based on the valuation currency, reserve currency vs. base currency, and mean of return prediction, historical vs. forward-looking.

***Key Words**: Small open economies, international reserves, cost of holding reserves, yield curve modeling, Bayesian inference, strategic asset allocation, reserve management framework*

***JEL Classification**: C11, C12, C61, C63, C87, C88, E43, E44, F21, F32, F33, F41, G11, G12*

INTRODUCTION

Two of the most interesting aspects discussed in relation to foreign exchange reserves are: What is meant by the "optimal" level of foreign exchange reserves and why they have been accumulated at a rapid phase?[1]. In addition, perhaps not in the same proliferation: Are the reserve accumulation

*Charith received his B.Sc from the University of Moratuwa and Master's from the University of California, Berkeley. He is currently working at the International Operations Department of the Central Bank of Sri Lanka. The contribution to this paper from the early work of Alex and Joachim [2009] is immense. The author would like to thank Dr.Joachim Coche and the BIS asset management team, Basel, Switzerland for their comments on the earlier draft and the Citi fixed income team and his colleagues for their insightful views. Last but not the least, he would also like to thank C.N. Wijayasekara and his department supervisors for facilitating his research and is also grateful to the Economic Research Department for providing him with a great opportunity to present this study at the 9th International Research Conference held in Colombo. The views expressed here are those of the author and not necessarily those of any institution. The author can be reached via charith@berkeley.edu.

[1] For example Jeanne [2007], Jeanne and Ranciere [2011] and Calvo et al. [2012]

trends and objectives of advanced economies vs. emerging-and-frontier markets the same or towards the same objective?[2].

Holding of reserves carries a significant cost, however, could be justified as an insurance premium to prevent the cost of a crisis; unless otherwise a proper level of reserves had not been maintained Jeanne [2007], Calvo et al. [2012]. Stemming from the above, the focal point of this study is to find out if there is a prudent top-bottom reserve asset allocation framework, especially for small open economies, that determines the: amount to be maintained as the return generation motive to reduce the cost of holding reserves, amount to be maintained in highly liquid form for the external sector stability, the reserve currency composition of the reserve, and the portfolio choice. To fill this gap, this study is based on the developments of international reserves of the selected frontier small open economies, and how a sustainable reserve management framework could be established. Therefore, the broad breakdown of the study is as follows. First, possible rationales for the currency compositions of reserve tranches will be discussed; this paper outlines the external sector trade exposure as a sustainable long-term guiding rule. Second, this paper outlines a framework that identifies how to minimize the cost of holding reserves via an optimal tranching of the reserve; Liquidity and Investment. The liquidity tranche will satisfy the needs arising for external sector stability–short term external debt servicing and domestic interventions. The investment tranche portfolio choice determines the amount of return out of the reserve for a desired risk level. The base model used for the investment tranche optimization, compares various extensions. The results uncovered shed light on the importance of the overall asset allocation framework and how the asset allocations could differ based on the reserve holder's perspective *i.e.* the central bank or other authority, and how it holds and manages the international reserves for the country.

Frontier, or Small Emerging Economies, by definition, are too small to be considered as an Emerging Economies, but more developed than the least developed markets. In the case of small open economies, there are two types of small open economies: Developed, which experience low volatile business cycles, and Developing, in contrast experience high volatile business cycles Guerron-Quintana [2013]. They are considered as small open due to the fact that their interest rates are determined by the international forces Schmitt-Grohe and Uribe [2012] and internationally vulnerable. Therefore, this study is confined to a cluster of developing small and open economies, that can be defined as small-emerging, or frontier.

When considering the history of rapid growth in foreign reserve accumulation, in emerging and small emerging economies, a point of pivotal could be recognized after the mid 1990's Aizenman and Lee [2007], Jeanne [2007]. According to the authors, the same pivotal point diverged the reserve accumulation trends in emerging and advanced economies–while advanced economies remained stable on reserve accumulation as a percentage of their GDP, emerging markets increased the levels of reserves to levels that cannot be

[2] See Goldberg et al. [2013] and Jeanne [2007]

explained by the traditional reserve adequacy theories [3] Jeanne [2007] In contrast, the following literature tries to identify key motives behind this rapid accumulation trend. The insurance motive hypothesis explains the reserve accumulation as a "flight to safety": The emerging and small emerging economies always wanted a buffer against the fear of a "sudden capital outflow", similar to they experienced during the Asian financial crisis in 1997 Aizenman and Lee [2005], Jeanne [2007], Eichengreen [2006], Obstfeld et al. [2010], Calvo et al. [2012], Stiglitz [2006]. The mercantilist hypothesis, on the other hand, argues this reserve accumulation trend as a result of motive to exploit reserves as a policy tool Aizenman and Lee [2005] to facilitate a export led growth, and itself is a "by product" of external trade activities. According to the mercantilist motive, accumulated reserves are a mean of a confidence to maintain a low exchange rate for trade competitiveness.

Reserve holding for both insurance and mercantilist motives are valuable ingredients to build-up the premise of this paper. A country could exploit the mercantilist motive, however, the reserve manager has to maintain a proper liquidity buffer to move for competitive lower exchange rates, and not let the currency to depreciate beyond the desired levels on speculations. This study refers to this buffer as the liquidity tranche and reserve manager cannot expect a higher return out of this tranche as its objective is to maintain the external sector stability; high liquid assets of this tranche should able to be liquidated at a lower cost in case of an emergency. However, the liquidity tranche should not be exceedingly large as there is a "cost" of holding reserves Jeanne and Ranciere [2011], Rodrik [2006], Calvo et al. [2012]. The cost component is mainly coming from the interest rate difference between the costs of short-term borrowing abroad as a relatively low credit rated sovereign, and the return that reserve managers could generate by investing those funds in high quality foreign assets. Therefore, return generation is important from a small-open economy's view point. According to the insurance motive hypothesis, if the cost component is the insurance premium against a sudden stop, the assets of the investment tranche should be allocated, and generate a return to minimize that cost within a given desirable risk budget.

Since a foreign currency reserve is a multi-currency asset pool, the mean or the perspective of identifying risk-return is a paramount concern Alex and Joachim [2009]. Therefore, the objective of this paper is to minimize the cost of insuring and mercantilist motives[4] via proposing a solid top-down framework, which is not only a rationale for a prudent asset allocation, but derived from the external sector.

This paper proceeds as follows: Section 1 discusses, recent observations of the reserves of small-emerging economies including a methodology that selects a sample of peer group small-emerging countries as a reference that guide towards the objective of the study. Further, Section 1 also discusses the concept of

[3] Conventional reserve adequacy ratios include: reserves to imports that ensures at least three months of imports are covered by the reserves Mendoza [2004], reserves to short-term external debt Greenspan [1999], reserves to broad money (M2) that inferred from the argument of broad money reflects the country's exposure to the withdrawal of assets Calvo [1996] etc.

[4] Insurance motive has a cost if excessive reserves are held and not invested optimally. Similarly, mercantilist motive urges reserve assets to be aligned with the external sector. Ultimately, a prudent return generation framework should neither under nor over invests the reserves and well aligned with the external sector dynamics.

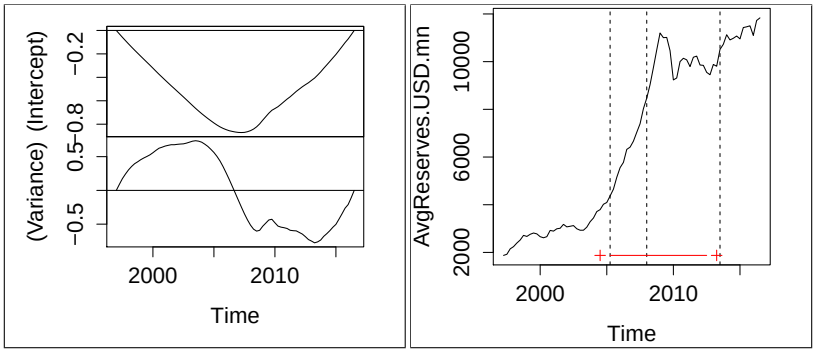
optimal level of reserves and how the concept would be deployed to assist the objective of this study. Therefore, Section 1 is a comprehensive analysis which lays the foundation for the asset allocation framework. Stemming from the results of the Section 1, Section 2 outlines the risk factor extraction methodology out of the Nelson-Siegel framework, and presents the results of the predicted yield curves of the small open economy reserve manager's investment destinations. Section 3 investigates asset allocations,optimization, and use of Bayesian techniques. Section 4 presents, interprets and discusses the results of the proposed asset allocation framework. Section 5 concludes the study by discussing the implications of the findings.

1. Dynamics of Foreign Exchange Reserves – Small Emerging Economies

The recent dynamics of reserve positions, especially of those small-emerging-and-open economies are somewhat cautious. The recent observations show that the reserve accumulation trend started in mid-1990, reached a peak in 2014 and started to stagnate.

1.1. Recent observations

To test the hypothesis, whether the countries in focus are entering in to a new era of reserve accumulation, this study starts from the Financial Times Stock Exchange(FTSE) [September 2014] classification of small emerging economies; the list consists of 25 countries. Subsequently, M-Fluctuation tests for parameter instability Zeileis and Hornik [2007] coupled with Hsu [1979], were conducted to detect shifts in parameters, with the intention of identifying structural breaks in reserve accumulation trends of the countries.Out of quarterly data–from 1996 to 2016 of average reserves–of those countries, optimal 4-segment partition were found as shown in the Figure 1(a) with three break points. Further, according to the results, the break dates of: 2005(Q2), 2008(Q1), and 2013(Q3), correspond to the peaks of the variance process shown in the Figure 1(b).



(a) M-Fluctuations Test *(b) Break-dates of the reserve accumilation*

Source:Author calculations

Figure 1: *M-Fluctuation test and the break-dates of the reserve accumulation*

As shown in the Figure 1(b), there is a most recent break in third quarter 2013, by an abrupt increase volatility of the average reserves of the small open economies. It can be assumed that the recent imbalances in global growth, which caused short term in-and-out liability capital flows made small emerging market international reserves fluctuate. Notwithstanding, asset price fluctuations of which reserve managers of those economies invested might have also contributed to the overall fluctuation. If the fluctuations confronted by either capital flows or by the asset price volatility, a prudent reserve management framework if in place, should minimize the impairment.

1.2. Defining a sample of small open economies

The purpose of defining a sample cluster of small open economies to conduct a peer analysis in a way that compare and contrast: What should be their reserve asset allocations based on their external sector dynamics.

As the initial sample, a group of 23 countries were selected, which could be assumed will satisfy the characteristics of: Small, Emerging, and Open. The first sample of countries are mainly from: Financial Times Stock Exchange(FTSE) [September 2014], Morgan Stanley Capital International (MSCI) [September 2014], S & P DOW Jhons [April 2016] country classification and selective subjective inclusions by the author. Accordingly, the sample was determined by Gross Domestic Production (GDP), trade openness, current account status, sovereign credit rating-that determines the rate at which the country borrows foreign currency from the international market, size of the international reserves, current account status ,and the short term-debt in foreign currencies. In addition, Brazil was added to the initial sample which satisfies the characteristics of open and Emerging.Therefore, the overall initial sample of countries are shown in the Table 6, with their GDP as of 2015, average percentage of trade as a percentage of GDP, average reserve levels, average current account balance, and the average short term borrowings as a percentage of total reserves. GDP and Trade to GDP are gauges that reflect the "size" and the dependency on international trade. Further, the current account deficit is a key determinant that would cause Sudden Stops of capital flows Calvo et al. [2008] and short term debt is a key determinant of measure the reserve adequacy in many IMF policy papers including International Monetary Fund [2011a].

Out of the initial subjective sample, K-means sampling is used to filter a mathematically significant cluster or a group of countries for further studies by solving the optimization Equation 1 by Lloyd [1980] algorithm:

$$arg\,min_S \sum_{i=1}^{k} \sum_{x \in S_i} \|x - \mu_i\|^2 \tag{1}$$

In this exercise, $S = \{S_1, S_2, S_3\}$ is a 3-dimensional vector, where atleast three clusters out of this country sample is expected;therefore, the $k = 3$. Since x is a set of d dimensional real vectors each with n observations, in case 5 dimensions are mentioned in Table 6 with 24 countries as the number of observations. Further, The

mean of points in S_i are given by μ_i. The results are shown in the Table 7 and Figure 6, show a 2-dimensional cross-section of the same; each cluster is shown in the colors of Red and Green. As expected Brazil was rejected from the test as an outlier while cluster 2 is fairly stable in terms of reserves, short term debt, and the current account position. Therefore, cluster 3 is in focus now as similar small-emerging-open-economies in-terms of the objective of the study, which consists of low reserves, GDP, with high international trade, current account deficit, and short-term debt.

1.3. Defining the currency composition of the reserves

As the next step, this study explores the trade dynamics of this selected cluster to provide a rationale for the determination of the reserve currency composition.The import and export ratios are the most robust determinant of the reserve holdings of the emerging economies over the past two decades [International Monetary Fund, 2011a].Therefore, the current account, trade component in particular, is introduced in this study as a main reference that could be used to determine the strategic long term currency composition of the proposed reserve asset allocation framework. Since these economies trade with different counterparts and different currencies, this study suggests the trade dynamics could be used as a determinant of the currency composition of the reserve.

The phenomenon could be illustrated by breaking down the Balance of Payments (BOP) of an economy. BoP records the overall balance of the country's foreign exchange inflows and outflows with the rest of the world. Therefore, BoP records net balance of the capital [5] and current accounts. According to the way it is defined, current account of the BoP is a natural inflow of foreign currencies from the small open economy, and hence should be a rationale determinant of the long term currency composition of the reserve asset allocation of the country. This nature of currency allocation is in accordance to the objective of this study to make sure the international reserves of the small open economy is in line with the external sector of the economy. However, when a small open economy current account deficit is not offset by the capital account [6], then the correction should occur through the foreign exchange reserve to make the BoP zero.

Therefore, the current account deficit must be met either by a capital surplus or by drawing upon reserves– or a combination thereof. However, from a reserve manager's view point, unlike the current account based currency composition impact on the reserve, the liability based impact is deterministic *i.e.* when the part of the reserve is borrowed, the reserve manger knows the future liability stream, in which currency and when to repay. Therefore, from a small open economy's view point, the unknown component is the natural impact on the currency composition caused via the current account. Hence, the overall currency composition of the reserve should be linked not only with the deterministic liability portion, but also with the trade related

[5] In this context, capital account is: capital plus financial account, according to the International Monetary Fund [1993] definition.
[6] Netting-off current account via capital account mainly represents the small open economy is increasing its foreign liabilities.

[7]activities with the rest of the world, thus can be defined as:

$$W_i = f(w_i^{std}, w_i^{trade}, \varepsilon_i) \tag{2}$$

According to Equation 2, W_i is the i^{th}currency weight of the reserve. Therefore, W_i is a function of the deterministic i^{th}currency short term debt exposure given by w_i^{std}, expected trade exposure to the currency, w_i^{trade},and ε_i –a random component.

The reserve manager has to do considerable approximations, when mapping Equation 2 into the reserve currency composition. Since w_i^{std} is known and should be repaid, w_i^{trade} should be determined to gauge the W_i or the i^{th}currency weight in the overall reserve over a predictive long-run horizon, also taking in to account the country specific dynamics from the other variables of the function.

Accordingly, Table 8 shows the 10 year average of trade, both import and export, exposure to the each reserve currency of EUR, GBP, USD, and JPY. Figure 7 shows the actual currency composition of the world's reserves—emerging and developing [8]. The most significant mismatch is in EUR, where the overall allocation of Figure 7 is much lower than the trade exposure to the EUR given in the Table 8. Second, the "other" component is much higher than the Figure 7.

From the trade perspective, most of these economies do trades under the currency category called "other". Further, a larger portion of this "other" component includes the trades with the neighbor countries and oil related trades. That being said, a reserve manager cannot hold the same proportion in such currencies given relatively the illiquid nature of investment assets in those currencies. So it is a rational approximation to map the other portion into USD, by assuming the shocks coming from the USD are regionally common and so is to Oil. Second, this author observed a significant portion of trade with China by most of these countries, which is growing as suggested by Mann-Kendall test Hipel K W [1994], Libiseller C [2002]. It is an interesting fact that IMF will separately identify the renminbi (RMB) in foreign exchange reserves starting October 1, 2016 International Monetary Fund [2015]. Therefore, according to the Equation 2, and it is a rational decision for countries under consideration to allocate a portion in renminbi in the investment tranche of this framework. However, the EUR mismatch remains questionable, and the most rational explanation is: reserve managers are prevented from investing in EUR given its negative yield environment. However, this could be a long run mismatch when the trade exposure to EUR is much higher than the reserve currency composition.

Notably, given w_i^{std} is not available and unique to the country, trade based reserve currency allocation is not a rule of thumb. However, which could be used as a guiding rule that determines the long run sustainable strategic reserve currency allocation.

[7] A further break-down of the Current Account includes both trade and the factor income, however this study assumes trade component is more representative related to a long term sustainable asset allocation. Therefore, factor income is considered as a part of ε_i of the Equation 2.

[8] From 2015 Q2 onwards IMF stopped classifying emerging and advanced economies, separately.

In summary, towards the ultimate objective of this study, that proposes a long-run top-down reserve asset allocation framework for the small emerging economies, this subsection discussed a rational that determines the currency composition of their reserves. For that, the first definition of Equation 2 and Figure 7 provides a gauge which is complemented by Table 8. According to the Table 8–except for Tunisia and Morocco, which are more EUR dependent in trade terms–it would be rational proportionate to maintain a higher portion of the reserve in USD and allocating rest of the reserve currencies trade proportionately, on top of the deterministic w_i^{std}. Notably, under normal circumstances, no currency conversion is recommended for liabilities or the borrowed portion of the reserve, assuming they will be paid back in the same currencies.

1.4. Defining the tranching methodology for the reserves

A reserve of a small emerging economy could be volatile based on three reasons. First, the cash-flow impact or the nature in which cash flows in and out in of the reserve. This is due to market interventions or due to repayment of liabilities, and linked to the external sector and the price stability objectives of a central bank as discussed under the Introduction. Second, due to the asset price volatility as a result of reserve assets are being exposed to the market fluctuations. Third, whether the valuation of the reserve assets is done in Reserve Currency , Base Currency or in Individual Currency perspective *i.e* whether or not the reserve assets are converted back to a common currency numeraire.

The first challenge, the cash flow dynamics, is handled by allocating an optimal liquidity buffer to the liquidity tranche.The most passive way of tranching is, for small open economies in particular, forecast the foreign debt payments and intervention, say for a year ahead, and allocating a very high buffer. The remaining will be channeled to the investment tranche. This approach is safe but costly; as this is a worst case based tranching based on the assumption, there will not be any borrowings from the government and the same intervention pattern will continue in the future. Instead, a proper tranching rule should be defined to minimize the "cost" of holding reserves discussed under the Introduction. Jeanne [2007] and Jeanne and Ranciere [2011] suggest that level of reserves should be an ideal balance of cost and benefits. As most of the discussed small emerging economies have a non investment grade sovereign credit rating[9], return on reserves is lower than the interest rate on long-term external liabilities; therefore, there is a conspicuous cost when the reserve managers try to invest the sovereign borrowings in high quality investment grade bonds. In contrast, the benefit of bearing the cost is: allow the government to smooth domestic absorption in a crisis.

Jeanne [2007] and Jeanne and Ranciere [2011] optimize the balance and finally derive[10] an optimal level

[9] According to the Fitch Ratings, Standard & Poor's, and Moody's, all the selected countries have foreign currency sovereign ratings below the prime investment grade, except for Morocco and Uruguay, which are at the lower medium grade.
[10] See Jeanne [2007] and Jeanne and Ranciere [2011] for the derivations.

of reserves, p_t^* that minimizes the holding cost as:

$$p_t^* = \frac{\lambda + \gamma - (1 - \frac{(r-g)\lambda}{1+g})(1 - p_t^{1/\sigma})}{1 - x_t(1 - p_t^{1/\sigma})} \tag{3}$$

Where λ is the ratio of short-term debt to GDP, γ is the output loss in the first period of the sudden stop; it was calculated γ as the average difference between the GDP growth rate the year prior to the sudden stop and the growth rate the first year of the sudden stop. Further r is the risk free rate and g is the average real GDP growth– in this case it was calculated over the past 30 years. Next the authors defined two parameters: δ_t as term premium (10 year U.S Treasury yield – Fed Fund Rate) and π_t as probability of sudden stops[11] given by the Equation 4 and 5 respectively.

$$\delta_t = x_t - \pi_t \tag{4}$$

$$p_t = 1 - \frac{\delta_t}{(1 - \pi_t)(\pi_t + \delta_t)} \tag{5}$$

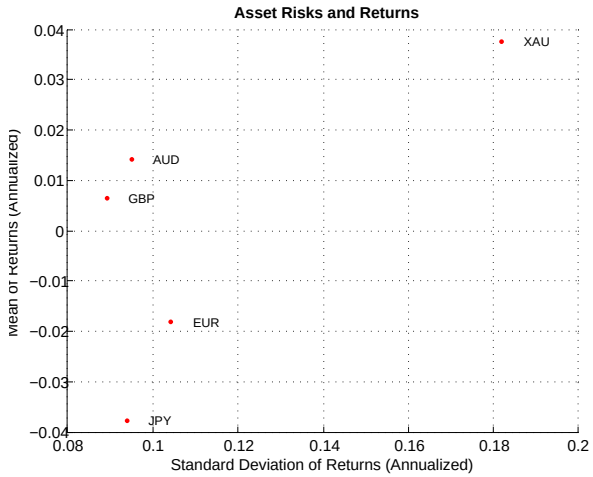
Accordingly, an optimal levels of reserves for the country sample is calculated in Table 9. It can be seen that the some countries are holding way excess reserves than the desired and incur a huge cost of holding reserves as described by Jeanne and Ranciere [2011]. Therefore, the excess should be in the high return generation portfolio; the investment tranche. However the question is: Should the whole reserve adequacy level given by Jeanne and Ranciere [2011] be in the low-return generation liquidity tranche? The answer is: Not necessarily, and should be based on the predictive cash in and outflows to the reserve. Next, the "sudden stop" models like Jeanne and Ranciere [2011] are not necessarily tranching models; they estimate how much reserve should a country have to face if the foreign inflows stopped suddenly. However, this study argues, Jeanne and Ranciere [2011] given good guidance for the tranching as well—as they are rooted in to the principle of cost-benefit analysis of holding a reserve.

1.5. Gold as a reserve asset

Gold has a historical value as a reserve asset and has been a hedge during the times of global risk, and when the value of the Dollar was losing grounds World Gold Council [2013]. However, due to valuation issues, sometimes Gold had not been taken in to account as a crucial determinant of reserve holdings–especially for small emerging economies Mwase [2012]. This study, however, decided to exclude Gold from the strategic level asset allocation due to its high volatility over the last 6 years of the post-crisis era *i.e* from 2009-2015, as shown in the Figure 2. Notably, compared to the other reserve currencies Gold is an outlier, and at the level

[11]To calculate the probability of sudden stops, 1 year sovereign credit transitional matrix of Standard & Poor's [2015], which provides the probability of a country being downgraded over the next 1 year assumed as a proxy.

of optimization, inclusion of Gold caused an undesirable influence over other asset classes.



Source: Author Calculations

Figure 2: *Volatility of Gold (from 2009-2016) relative to the other reserve currencies*

2. Risk Factor Identification and The Yield Curve Construction

2.1. Risk factor identification

In this section, this author considers small open economy reserve mangers' investments are in investment grade government securities denominated in the major currencies identified in the Section 1.3, where the external sector trade exposure had been mapped to the main reserve currencies identified under the IMF [2016]. Same as any fixed income security, government securities carry a higher market risk caused by the yield curve movement. In addition, depending upon the reporting framework, there could be a currency risk, which will be discussed later.

Latent factors, which drive the yield curve dynamics Nelson and Siegel [1987], Litterman et al. [1991], Dai and Singleton [2000], Diebold and Li [2006] expose fixed income investments of the reserves of the small open economies under focus to market risk, and therefore contributing to overall volatility of their reserves. This study identifies latent factors of Level, Slope, and Curvature from the first foundation work of Nelson and Siegel [1987]. Therefore, as shown in the Equation 6, the yield of an investment is measured by $y_t(\tau)$– yield on a zero coupon bond maturing at time τ.

$$y_t(\tau) = \beta_{0_t} + \beta_{1_t}\left(\frac{1-e^{-\lambda_t\tau}}{-\lambda_t\tau}\right) + \beta_{3_t}\left(\frac{1-e^{-\lambda_t\tau}}{-\lambda_t\tau} - e^{-\lambda_t\tau}\right) + \vartheta_t(\tau) \tag{6}$$

Here in the Equation 6, $\beta_{0_t}, \beta_{1_t}, \beta_{3_t}$ represents the latent factors in the same order they were above-introduced, and λ_t is a parameter and error term $\vartheta_t(\tau)$ is a distribution with standard deviation $\sigma_t(\tau)$; factor loadings are independent of t. Further, the Level factor captures the mean yield level when the maturity goes to infinity, and corresponds to a parallel shift of the yield curve. Slope is the driver that determines short term yield movement comparative to the long end driver, while the Curvature captures almost the opposite movement of which Slope factor captures. Nelson and Siegel [1987] latent factors, Level, Slope, and the Curve vary with the time as the yield curve vary with time; this is in fact is the market risk of a reserve manager's investments and cause investments to be volatile.

2.2. Methodology and results: Yield curve construction and factor prediction

Stemming from the Nelson and Siegel [1987] foundation, the objective of this exercise is to construct the predictive yield curves for the U.S, Germany, Japan, U.K, China, Australia, and Canada fixed income securities. Therefore, one can assume, based of the facts discussed thus far, small-emerging economies invest their foreign exchange reserve holdings of USD, EUR, JPY, GBP, CNY, AUD and CAD in fixed income securities. The predicted yield curves for the next 5 years from 2016 complement the final step of this study–a strategic asset allocation framework.

As the literature discussed, βs as risk are risk factors and the matrix β_t is defined as,

$$\beta_t = \begin{bmatrix} \beta_{1_t} \\ \beta_{2_t} \\ \beta_{3_t} \end{bmatrix} \tag{7}$$

Therefore, the predictive autoregressive[12] model is:

$$\beta_{\mathbf{t}} = A + B\beta_{\mathbf{t-1}} + \Sigma_{\vartheta_t} \tag{8}$$

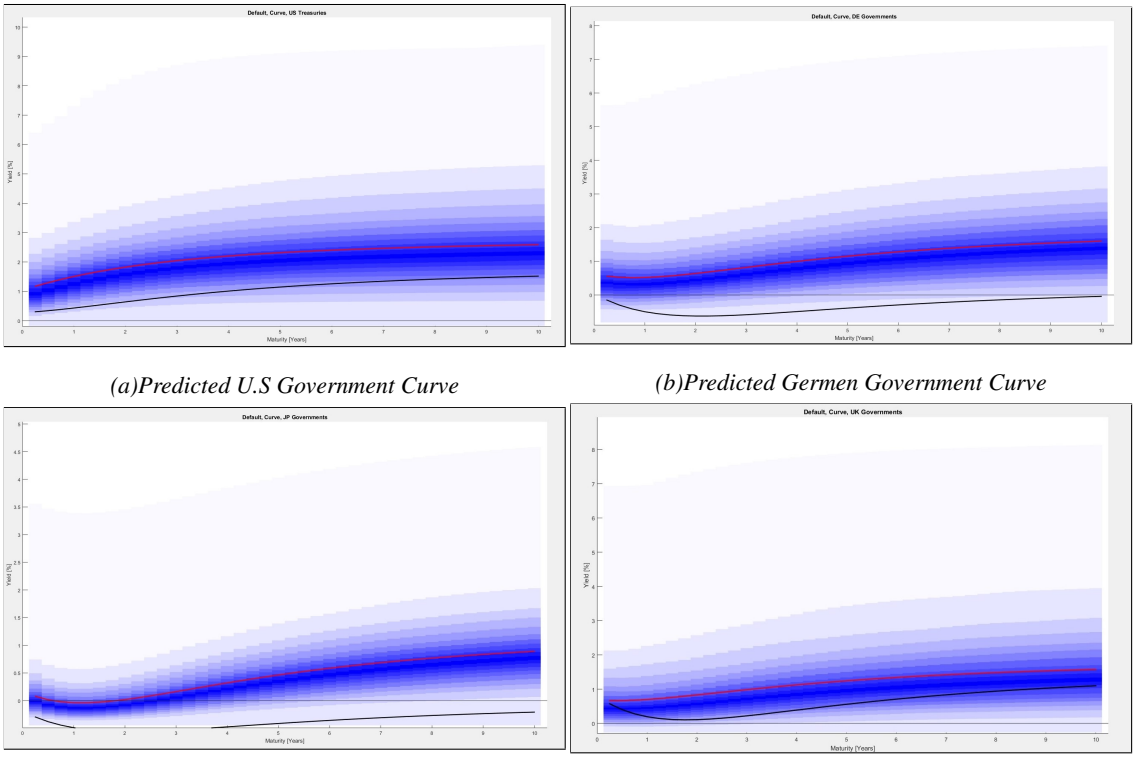
where the factor loading matrices A, B, and Σ are calculated using the empirical risk factor data of the each yield curve. Next, the factors are predicted according to Equation 8 and combined back to obtain predictive yield curve for each country. The predictive factors, for all the yield curves, are shown in the Figure 8. This figure shows the extracted $\beta_{0_t}, \beta_{1_t}, \beta_{2_t}$ for each and every government bond yield curve discussed above. In addition, the figure shows the other reserve currency movements, with respective to the USD[13]; the "currency

[12]This process assumes risk factors, individually, are long run mean reverting. See Equation 14 for the derivation and see Campbell and Chan [2001], Campbell et al. [2002] for details.

[13]In USD base currency perspective, the risk-return trade-off is coming not only from the β risk factors, but from the currency movements with respective to USD as well. However, individual currency assumes no currency conversion. This will be discussed in detail later under the Section 4.

perspectives" will be discussed at a later print of this paper.

The predicted U.S Treasury yield curve, German government bond yield curves, Japanese Government Curve, and the U.K Government Curves–four major reserve currencies– are shown in the Figure 3. In addition, this author has calculated the predicted yield curves for China, Australia,and Canada.



(a)Predicted U.S Government Curve

(b)Predicted Germen Government Curve

(c)Predicted Japanese Government Curve

(d)Predicted U.K Government Curve

Source:Author calculations based on data from Bloomberg using the factor-based model

Figure 3: *Predicted yield curves*

One common feature associated with all the predicted yield curves is that, they are expected to steepen over the next five year horizon. German and Japanese curves–where the front end is in the negative territory–are also expected to steepen via an upward shift. This is a notable fact from a small-emerging country reserve manager's perspective when determining the asset allocation and the optimization.

3. Asset Allocation: Optimization and Use of Bayesian Techniques

As the generalized currency compositions and risk factors –with respective yield curves–have been identified, the selection of assets and the optimization are the next steps that lie ahead.

3.1. Selection of assets

As mentioned in the Section 2.2, this study assumes small emerging economy reserve manager would insist in government securities of the United States(U.S), Germany, United Kingdom (U.K), Japan, China, Australia and Canada–denominated in USD, EUR, GBP, JPY, CNY, AUD and CAD, respectively–as the strategic level asset allocation for the investment tranche. Further, this author assumed USD as the main reserve currency and the maturities vary from 3 months to 10 years. For other reserve currencies, the author decided to remain in the shorter maturity given: adding duration risk for the overall risk budget, by moving in to higher duration benchmarks cause an amplified effect when the reserves are valued in a common numeraire[14]. A summary of the selected assets are given in the Table 10.

3.2. Portfolio selection and optimization

Optimization is done ascertain to how much in each asset– mentioned in the Section 3.1– should be invested for the risk-return to be optimized. The Modern Portfolio Theory (MPT), is one way of attacking the problem. In financial economics, and statistical models and methods for financial markets Markowitz (Markowitz [1952], Stuart and Markowitz [1959]) is widely regarded as his first contribution to mean-variance based portfolio theory. Markowitz's introduction, and subsequent criticisms helped the model to have many extensions. Notably, the classical Markowitz approaches assume that returns are normally distributed and the researcher has no prior knowledge on the asset returns. In this context, however, prior knowledge on asset returns and risk factors is used to predict the posterior distribution, and therefore overcome from the above-mentioned drawback in the Markowitz approach. The Bayesian inference is a powerful way of addressing the issue by deriving a posterior distribution with the help of a prior distribution Zellner and Chetty [1965], Zellner [1971], Jorion [1986], Jose and Smith [1994], Avramov and Zhou [2010].

3.3. Methodology

The first Markowitz [1952] framework suggests that there is an expected return matrix of μ of portfolio w with n number of assets. Therefore, let the return matrix be $\mu_{px} = w'\mu$ and the risk matrix be $\sigma^2_{p,w} = w'\Sigma w$, where Σ is the covariance matrix of the asset returns which are normally distributed. Therefore, the objective function be:

$$U(w) = \mu_{pw} - \sigma^2_{p,w} = w'\mu - \frac{\lambda}{2} w'\Sigma w \tag{9}$$

[14]When the other reserve currencies are invested in higher duration securities, two factors contribute to the overall volatility of the reserve: Duration risk as well as the Currency risk. This author does not see having a leveraged risk budget, from currency as well as from the bond duration, is a sustainable approach from the overall reserve volatility view point.

where λ is a risk aversion parameter here. The first order conditions (FOC) gives the optimal weights that maximizes the utility function :

$$U(w^*) = \frac{1}{2\lambda}\mu'\Sigma^{-1}\mu \tag{10}$$

In this context, both μ and Σ assume no prior knowledge and gives only a point estimation. However, when it comes to the Bayesian approach, the optimal weight is calculated based on the input modification based on the prior knowledge of the returns as comprehensively illustrated by Avramov and Zhou [2010]. Therefore, the optimal weight of the Bayesian approach is given by:

$$w^*_{Bayes} = \underset{w}{\operatorname{argmax}} \int_{R_{t+1}} \tilde{U}(w)p(R_{t+1}|\Phi_T)dR_{t+1} \tag{11}$$

$p(R_{t+1}|\Phi_T)$ is the return at $t+1$ conditional on the data available at time t,Φ_T. Further, $\tilde{U}(w)$is the utility at time t. Compared to the classical Markowitz approach, the Bayesian inference maximizes expected utility based on conditional and predictive distributions given as:

$$w^*_{Bayes} = \underset{w}{\operatorname{argmax}} \int_{R_{t+1}} \int_{\mu} \int_{\Sigma} \tilde{U}(w)p(R_{T+1},\mu,\Sigma|\Phi_T)d\mu d\Sigma dR_{t+1} \tag{12}$$

$$p(R_{T+1},\mu,\Sigma|\Phi_T) = p(R_{T+1}|\mu,\Sigma,\Phi_T)p(\mu,\Sigma|\Phi_T) \tag{13}$$

where $p(\mu,\Sigma|\Phi_T)$ is the posterior density, and inputs Σ,μ are modified for the each time $t+1$ based on the information available at time t.

3.4. Constrained Optimization

The optimizer provided concentrated higher weights on certain asset classes which were beyond the ranges discussed under the Section 1.3, due to a short-come in practical optimization known as the "corner solutions" problem Ruiz-Torrubiano and Suárez [2015]. Therefore, in this context, to be consistent with the Section 1.3, constraints were put in place for China, Japan, U.K, Australia and Canada exposures *i.e* during the optimization, solutions will contain within 10% range for the mentioned countries–and, not beyond. However, no constraints were put on the U.S.A and Europe, the major trading partners of the selected cluster.

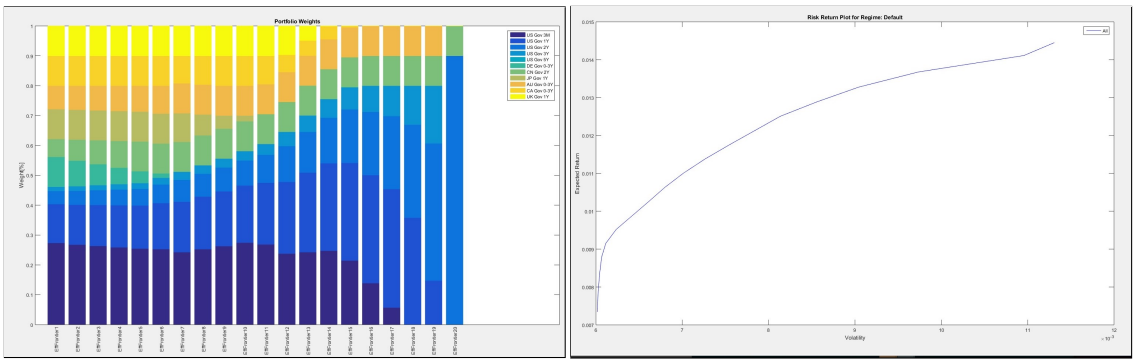
4. Results

As lightly mentioned previously, in Section 2.2, there are two ways of looking at asset returns: individual or base currency, and conversion to a common base numeraire such as USD. There is also a third way of looking at the asset returns in domestic currency. Similarly, based on the valuation convention, Σ or the covariance

matrix of the strategic asset allocation in optimization process changes [15]— so does the final asset allocation. Therefore, the valuation framework matters, and two optimizations were done to illustrate this fact.

4.1. Individual or reserve currency perspective

Figure 4(a) shows the optimal strategic asset allocation, if the reserve managing authority of the small open economy would like to tolerate the other reserve currency such as EUR, GBP etc. volatility with respective to the USD. However, the reserve management authority seeks a sustainable asset allocation that is optimally allocated against the expected yield curve fluctuations of the individual currency. This regard, 20 possible asset allocation combinations were calculated: the leftmost asset allocation of Figure 4(a) represents the minimum volatility portfolio, and as the reserve manager moving towards the right side, a riskier budget is allocated seeking higher return. Figure 4(b) shows the corresponding mean-variance efficient frontier.



(a)Strategic asset allocation *(b)Markowitz efficient frontier*

Source:Author calculations based on data from Bloomberg using factor-based model

Figure 4: *Strategic asset allocation and the corresponding efficient frontier:Individual currency returns view point*

The tabular presentation of the Figure 4 is given in Table 1 and the corresponding risk-return characteristics are given in Table 2. The minimum variance portfolio (EF1) is expected to allocate more on short-term US securities. Further, the minimum variance reserve asset allocation, local currency perspective, is expected to generate a 0.7% return at a 0.6% volatility over the predicted five year horizon. That being said, the reserve manger can take a higher duration risk and go for the EF20 where the main concentration is on US 2 year bonds and the Chinese 2year government bonds. The expected risk-return is 0.14% and 1.1%, respectively.

[15]See Section 3.3

Table 1: *Asset weights first and last five efficient frontiers: Individual currency returns view point*

Asset class weights											
	EF1	EF2	EF3	EF4	EF5	...	EF16	EF17	EF18	EF19	EF20
US Gov 3M	0.27	0.27	0.26	0.26	0.25	...	0.14	0.06	0.00	0.00	0.00
US Gov 1Y	0.13	0.13	0.14	0.14	0.14	...	0.36	0.40	0.36	0.15	0.00
US Gov 2Y	0.04	0.05	0.05	0.05	0.06	...	0.21	0.24	0.31	0.46	0.90
US Gov 3Y	0.01	0.02	0.02	0.02	0.02	...	0.09	0.10	0.13	0.19	0.00
US Gov 5Y	0.00	0.00	0.00	0.00	0.00	...	0.00	0.00	0.00	0.00	0.00
US Gov 10Y	0.00	0.00	0.00	0.00	0.00	...	0.00	0.00	0.00	0.00	0.00
DE Gov 0-3Y	0.10	0.09	0.07	0.05	0.04	...	0.00	0.00	0.00	0.00	0.00
CN Gov 2Y	0.06	0.07	0.08	0.09	0.10	...	0.10	0.10	0.10	0.10	0.10
JP Gov 1Y	0.10	0.10	0.10	0.10	0.10	...	0.00	0.00	0.00	0.00	0.00
AU Gov 0-3Y	0.08	0.08	0.08	0.08	0.09	...	0.10	0.10	0.10	0.10	0.00
CA Gov 0-3Y	0.10	0.10	0.10	0.10	0.10	...	0.00	0.00	0.00	0.00	0.00
UK Gov 1Y	0.10	0.10	0.10	0.10	0.10	...	0.00	0.00	0.00	0.00	0.00

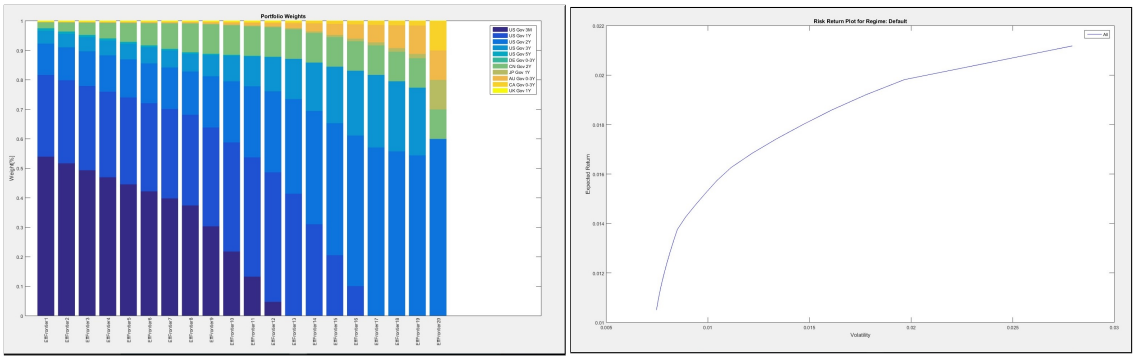
Table 2: *Expected risk-return : Individual currency returns view point*

	EF1	EF2	EF3	EF4	EF5	...	EF16	EF17	EF18	EF19	EF20
Expected return	0.007	0.008	0.008	0.008	0.009	...	0.013	0.013	0.014	0.014	0.014
Volatility	0.006	0.006	0.006	0.006	0.006	...	0.009	0.009	0.010	0.011	0.011
VaR return (95%)	0.001	0.001	0.001	0.001	0.002	...	0.002	0.002	0.001	-0.001	-0.001
CVaR return (95%)	0.000	0.000	0.000	0.000	0.000	...	0.000	-0.001	-0.002	-0.004	-0.004
VaR return (99%)	-0.001	-0.001	-0.001	-0.001	0.000	...	-0.001	-0.002	-0.003	-0.006	-0.006
CVaR return (99%)	-0.002	-0.002	-0.002	-0.002	-0.001	...	-0.003	-0.004	-0.006	-0.010	-0.010
Prob. of neg. returns	0.032	0.026	0.020	0.016	0.014	...	0.019	0.024	0.036	0.059	0.061
Duration	1.016	1.031	1.045	1.058	1.072		1.443	1.566	1.733	2.006	2.000

4.2. A common base numeraire perspective

Figure 5(a) shows the optimal strategic asset allocation, if the reserve managing authority of the small open economy would like to report the asset volatility, by taking in to account as a result of both currency volatility as well as interest rate volatility. Therefore, there are two factors that are driving the overall volatility of investments. Other reserve currency such as EUR, GBP etc. volatility with respective to the USD, as well as individual currency expected yield curve fluctuations. Similarly, 5(b) shows the corresponding efficient frontier and the Tables 3 and 4 show the tabular representation of the portfolio allocations and the risk-return from the USD view point.

Compared to the local return approach discussed in Section 4.1, USD numeraire valuation approach is vulnerable to the rest of the reserve currency volatility against the USD. Therefore, most of the risk budget should be allocated to tolerate the currency volatility. This could be clearly seen in the minimum variance portfolio, where 97% of the reserve should be allocated to USD denominated assets to have a 0.7% minimum volatility.



(a)Strategic asset allocation *(b)Markowitz efficient frontier*

Source:Author calculations based on Bloomberg using factor-based model

Figure 5: *Strategic asset allocation and the corresponding efficient frontier:Base USD returns view point*

Table 3: *Asset weights first and last five efficient frontiers: USD returns perspective*

Asset class weights											
	EF1	EF2	EF3	EF4	EF5	...	EF16	EF17	EF18	EF19	EF20
US Gov 3M	0.54	0.52	0.50	0.47	0.45	...	0.00	0.00	0.00	0.00	0.00
US Gov 1Y	0.28	0.28	0.28	0.29	0.29	...	0.11	0.00	0.00	0.00	0.00
US Gov 2Y	0.11	0.11	0.12	0.12	0.13	...	0.51	0.57	0.56	0.55	0.60
US Gov 3Y	0.04	0.05	0.05	0.05	0.05	...	0.21	0.24	0.23	0.22	0.00
US Gov 5Y	0.01	0.01	0.01	0.01	0.01	...	0.00	0.00	0.00	0.00	0.00
US Gov 10Y	0.00	0.00	0.00	0.00	0.00	...	0.00	0.00	0.00	0.00	0.00
DE Gov 0-3Y	0.00	0.00	0.00	0.00	0.00	...	0.00	0.00	0.00	0.00	0.00
CN Gov 2Y	0.02	0.03	0.04	0.05	0.06	...	0.10	0.10	0.10	0.10	0.10
JP Gov 1Y	0.00	0.00	0.00	0.00	0.00	...	0.01	0.01	0.01	0.02	0.10
AU Gov 0-3Y	0.00	0.00	0.00	0.00	0.00	...	0.05	0.06	0.08	0.10	0.10
CA Gov 0-3Y	0.00	0.00	0.00	0.00	0.00	...	0.01	0.01	0.02	0.02	0.10
UK Gov 1Y	0.00	0.00	0.00	0.00	0.00	...	0.00	0.00	0.00	0.00	0.00

Table 4: *Expected risk-return : USD returns perspective*

	EF1	EF2	EF3	EF4	EF5	...	EF16	EF17	EF18	EF19	EF20
Expected return	0.010	0.011	0.011	0.012	0.012	...	0.018	0.018	0.019	0.020	0.021
Volatility	0.007	0.008	0.008	0.008	0.008	...	0.015	0.016	0.018	0.020	0.028
VaR return (95%)	0.002	0.002	0.002	0.002	0.002	...	-0.005	-0.007	-0.009	-0.012	-0.024
CVaR return (95%)	0.001	0.001	0.001	0.001	0.001	...	-0.010	-0.013	-0.016	-0.020	-0.035
VaR return (99%)	0.000	0.000	0.000	0.000	0.000	...	-0.013	-0.017	-0.020	-0.025	-0.043
CVaR return (99%)	-0.001	-0.001	-0.001	-0.001	-0.002	...	-0.018	-0.021	-0.026	-0.030	-0.052
Prob. of neg. returns	0.012	0.010	0.011	0.011	0.012	...	0.103	0.120	0.140	0.159	0.235
Duration	0.837	0.872	0.909	0.946	0.983		2.072	2.194	2.180	2.160	1.832

One notable fact is when the reserve risk-returns are calculated in USD terms as shown in Table 3 and 4

compared to Table1 and 2 , in the case of local returns, the reserve manger has limited room to allocate other reserve currencies to the reserve if the reserve managing authority is risk averse to having a higher expected reserve volatility. However, in each efficient frontier the allocations provide a higher return when the currency volatility is factored in to the reserve manager's performance. For example, out of the frontiers chosen, for the next five year horizon the strategic asset allocations can produce an expected return form 1% to 2.1%; reserve currency perspective, it was 0.07% to 0.14%. Similarly, from the risk perspective, the volatility can vary from 0.7% to 2.8% depending upon which asset allocation is selected out of the frontier compared to 0.6% - 1.1% of the reserve currency view point.

4.3. Predicting future allocations based on historical returns

The interest rates or the yields of the advanced economies, that the small open economy reserve managers invest their reserves, have been deteriorating continuously over the past, and some such as Japan and Germany reached negative territory. This resulted in higher capital gains on fixed income investments of the small emerging reserve managers. Section 2.2 showed this trend will not be the last for the next 5 year horizon, but the yield curves are likely to steepen. Therefore, this study does not recommend to predict risk-returns based on the historical returns. However, for the purpose of comparison the same optimization exercise of the Section 4.2 was done, however using historical returns from 2003—instead of predictive or futuristic returns.

Table 5: *Annualized historical returns based optimization:USD numeraire*

	EF1	EF2	EF3	EF4	EF5	...	EF16	EF17	EF18	EF19	EF20
Average return (arithmetic)	0.0176	0.0184	0.0192	0.0200	0.0209	...	0.0333	0.0349	0.0358	0.0366	0.0355
Average return (geometric)	0.0170	0.0177	0.0184	0.0192	0.0199	...	0.0332	0.0350	0.0362	0.0374	0.0382
Volatility	0.0185	0.0189	0.0193	0.0198	0.0203	...	0.0338	0.0367	0.0405	0.0447	0.0680
Sharpe ratio	0.2418	0.2784	0.3139	0.3478	0.3797	...	0.5942	0.5914	0.5581	0.5254	0.3277
VaR return (95%)	0.0011	0.0011	0.0012	0.0007	0.0001	...	-0.0099	-0.0136	-0.0196	-0.0271	-0.0704
CVaR return (95%)	0.0007	0.0006	0.0002	-0.0003	-0.0010	...	-0.0160	-0.0202	-0.0281	-0.0361	-0.0861
VaR return (99%)	0.0004	0.0001	-0.0001	-0.0009	-0.0018	...	-0.0182	-0.0217	-0.0288	-0.0381	-0.0980
CVaR return (99%)	0.0001	-0.0002	-0.0007	-0.0012	-0.0019	...	-0.0250	-0.0308	-0.0407	-0.0508	-0.1052
Prob. of neg. returns	0.0000	0.0067	0.0134	0.0336	0.0470	...	0.1879	0.1946	0.2081	0.2081	0.2886
Duration	0.8373	0.8725	0.9090	0.9462	0.9833	...	2.0719	2.1936	2.1797	2.1602	1.8320

As the Table 5 returns based asset allocations generate a higher increase— compared to the 1.00% to 2.1% of Section 4.2— in returns from 1.76% to 3.55%, if the input returns were futuristic for the same exercise.

4.4. Domestic currency perspective

With reference to Equation 10 of Section 3.3 and Section 4.3, it was seen that when the return matrix changes, the expected risk-returns are different. Another perspective of looking at this problem is the Reserve Currency Perspective—converting the reserve manager's returns in to the domestic currency, and then optimize. It is an apparent fact the expected returns and the frontiers will be completely different given the return matrices

are different. That being said, for this exercise to be precise, the returns should be calculated in a way that the yield spread of the reserve manger's investments are compared to the domestic yield curve. It is a one comprehensive model that deals with not only with the foreign exchange reserve returns, but with all the external aspects including the exchange rate. However, the countries in the selected cluster do not have a long history of their domestic yield curves to do this in-depth calculation and this perspective is not eloquent within the tools available.

4.5. Review of results

After considering all of the above, the results of this study show that the framework in which the reserve valuation done is important for the reserve manager to frame a particular asset distribution framework. In the individual currency returns perspective, the reserve manager has more room to invest in other reserve currency and allow economy to be in line with a sustainable current account based currency allocation. In this context, the reserve manager may have a unity of command that his responsibility which is towards the interest risk of individual currencies. Noteworthy, to maintain that the reserve currency perspective is only a reporting mechanism to be in line with the long term trade exposures.

In contrast, if reserve reporting and performance measurement are in a common base currency such as USD, more uniformity can be ensured. Therefore, both the reserve manager and the reserve management authority are in the same task of optimizing the risk-returns in USD terms. However, when the trade exposures are in other currencies, the reserve management authority may not be able to allocate sufficient other reserve currencies without taking a higher volatility risk. It was seen that, risk budget contribution from the currency volatility is much higher compared to the interest rate volatility. Therefore, when the reporting and the reserve return generation is done in a common currency numeraire such as USD, coupled with proportionate external sector trade exposures in other reserve currencies, it is inevitable that the reserve management authority has to move in to a higher risk-return frontier.

However, reserve asset allocations based on USD base currency returns would be favorable for countries like Jamaica, Guatemala , Nicaragua, Dominic Republic and El Salvador as their main trade exposures are in terms of USD as shown in the Table 8. Therefore the mentioned countries, in their reserves, can have a higher percentage in USD that does not contribute to the currency risk budget. However, for countries like Tunisia, Morocco, Cameron and Cyprus, USD perspective may not be the ideal allocation as their trade exposures are mostly in non USD counterparts. For mid USD trade exposed countries such as Sri Lanka, Kenya and Costa Rica both approaches would would work, but the risk-return reporting framework should be framed clearly—therefore, the reserve manager and the reserve managing authority both can collaborate together, consistently, to move in to a sustainable long run return generation plus a sustainable external sector.

4.6. The currency allocation for the liquidity tranche

This study discussed two types of tranches; liquidity[16] and investment tranche, where optimization was done for the investment tranche. That being said, there will be a considerable portion of money in the liquidity tranche as well, and will be in reserve currencies that need to intervene to the market and to repay the debt obligations. As discussed in the Section 1.3 and 1.4 this amount is subjective to the internal short term cash-flow dynamics. One can optimize this buffer for returns using a micro-based model like the Melo and Bilich [2011], but it is a completely different exercise compared to the strategic asset allocation model.

5. CONCLUSION

This study provides comprehensive evidence on foreign exchange reserves of small open economies undergoing a challenging period requiring a sustainable reserve asset allocation framework. Further, this author uncovers a prudent approach that these economies may follow in deciding their reserve currency compositions and amounts to be allocated to the low return-high liquid tranche and more risk-return based investment tranche. That being said, given the strong idiosyncratic factors that affect the reserve dynamics of these countries, reserve currency compositions and tranche amounts are difficult to define via rule of thumb. Yet, this paper shows that international trade and the concept of cost of holding reserves could be used as guides for the objective. It also showed that trending exposure to the China and IMF inclusion of Chinese renminbi in to the SDR basket could be channeled to strategic investments.

Once the investment tranche is outlined, and the expected yield curves using Nelson-Siegel risk factors have been computed for major reserve currencies, the five year investment horizon ahead of the reserve manager of the selected cluster of the small open economies will be uncovered. The study found that yield curves of the investment destinations will likely steepen with a parallel upward shift. This is a challenging investment environment given that the reserves of the small open economies will not be able to benefit from the capital gains from their fixed income investments. Thereby, asset allocation choices were presented with the expected risk-return characteristics by optimizing the selected portfolios using Bayesian optimization in Markowitz framework. Further, this optimization was done for different risk-return matrices to show how the perception differences could lead to different asset allocations—where the main perceptions were valuation of all the reserve in a common base currency numeraire, and valuation of foreign reserve assets individually via the corresponding reserve currency without any currency conversion.

This top-down approach, from the external sector to the reserve asset allocation, urges for a more structured framework for the selected small open economies by providing author calculated choices. In the

[16]Not to be confused with the terminology, and this section discusses about the very bottom high liquid layer of a reserve subjected to often cash in-and-out flows, in the course of facilitating the external sector needs such as foreign debt servicing and interventions.

event the reserve managing institution is more favorable on reporting the returns of the reserve in individual currency and the reserve volatility due to the currency volatility is not material, the reserve manager's asset allocation strategy should be in line with it. In contrast, the reserve managing authority would consider the overall volatility of the reserve in a base numeraire such as USD; therefore, the currency volatility is factored in to the risk-return of the reserve manager. In this context, regardless of the external sector exposure, the reserve manager is required to maintain a very high percentage of USD to absorb the risk coming from the currency volatility. Therefore, whenever the reserve manager adds other reserve currencies, it can be seen that both risk and return increases remarkably. In further investigating underlying forces driving the small open economy reserve dynamics, this author hopes facts found are useful for future research, especially should one attempt to find a solution for the same problem via a micro-level, cash flow based bottom-up approach.

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APPENDICES

A. Proofs

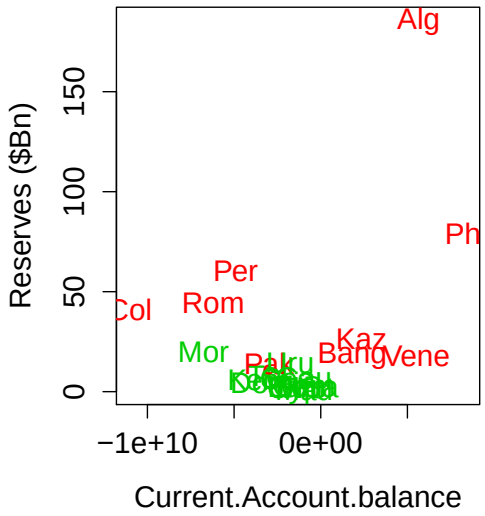
Let an autoregressive process $AR(1)$:

$$Y_t = a_0 + a_1 Y_{t-1} + \varepsilon_t : \varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) \tag{14}$$

$$Y_t = a_0 + a_1 Y_{t-1} + \varepsilon_t$$
$$where\ \varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$$
$$we\ take\ the\ expectation\ of\ both\ sides:$$
$$\mathbf{E}[Y_t] = \mathbf{E}[a_0] + \mathbf{E}[a_1 Y_{t-1}] + \mathbf{E}[\varepsilon_t]$$
$$\mathbf{E}[Y_t] = \frac{a_0}{1 - a_1} : as\ \mathbf{E}[Y_t] \approx \mathbf{E}[Y_{t-1}]$$

Therefore, if $|a_1| < 1$, Equation 14 has a mean level to which it reverts, and each-and-every risk factor satisfies these conditions.

B. List of Tables and Figures of the Section 1



Source: Author Calculations

Figure 6: *A 2-dimensional cross-sectional view of the K-means solution*

Table 6: *Initial Sample*

Country	GDP (USD million)	Trade (% of GDP)	Reserves (USD Billion)	C/A balance(USD million)	S/T Debt (% of Reserves)
Algeria	167,000	63.63	184.8841361	5,649	0.82
Bangladesh	195,000	46.3	17.8719677	1,815	20.44
Brazil	1,770,000	25.63	360.3465568	-71,674	14.24
Cameroon	29,198	49.57	3.351178585	-961	4.25
Colombia	292,000	37.7	40.99307084	-11,011	26.12
Costa Rica	51,106	74.49	6.799836046	-1,991	45.17
Cyprus	19,319	99.18	0.904482178	-1,554	0
DominicanRepublic	537	56.62	4.496426562	-3,219	45.96
El Salvador	25,850	71.95	2.778243467	-1,022	49.2
Guatemala	63,794	58.55	6.995809619	-963	0
Jamaica	14,005	83.4	2.296713864	-1,318	56.66
Kazakhstan	184,000	63	26.39068334	2,348	32.69
Kenya	63,398	51.28	6.121779191	-3,892	29.48
Madagascar	9,980	73.15	0.913868819	-953	45.14
Morocco	100,000	79.71	19.92795051	-6,763	21.46
Nicaragua	12,692	106	2.108172962	-981	76.25
Pakistan	270,000	33.34	14.23484279	-2,988	32.88
Peru	192,000	48.88	60.35550154	-4,903	12.68
Philippines	292,000	60.23	79.13748903	8,393	17.05
Romania	178,000	80.27	44.387251	-6,202	34.6
Sri Lanka	82,316	49.26	7.219967923	-2,407	71.04
Tunisia	43,015	104.8	7.887794067	-3,104	67.86
Uruguay	53,442	49.72	14.67435843	-1,755	0
Venezuela	381,000	54.28	18.29191325	5,524	0

Source: World Bank

Note: Trade, Reserves, and Current Account balances are five year averages from 2011 to 2014

Table 7: *Clustered Sample*

	Cluster means		
Cluster name	1	2	3
GDP (USD million)	1,770,000	239,000	40,618
Trade (% of GDP)	26	54	72
Reserves (USD Billion)	360	54	6
Short.Term.debt (% of Reserves)	14	20	37
C/A balance(USD million)	-71,674	-153	-2,206

	K-means clustering with 3 clusters of sizes 1, 9, 14		
Country Name	Cluster	Country Name	Cluster
Brazil	1	Kenya	3
Algeria	2	Morocco	3
Kazakhstan	2	El Salvador	3
Bangladesh	2	Costa Rica	3
Venezuela	2	Madagascar	3
Pakistan	2	Cameroon	3
Philippines	2	Uruguay	3
Colombia	2	DominicanRepublic	3
Romania	2	Cyprus	3
Peru	2	Guatemala	3
Sri Lanka	3	Jamaica	3
Tunisia	3	Nicaragua	3

Source: Author Calculations

Table 8: *Average trade exposure:country wise from 2005-2014*

	Euro Area	U.K	U.S.A	Japan	China	Aus & CA	Others	USD*
Sri Lanka	12.7%	5.8%	10.4%	2.4%	6.4%	1.9%	57.3%	67.7%
Tunisia	63.0%	2.5%	3.3%	0.8%	0.3%	3.0%	24.4%	27.6%
Kenya	18.5%	5.7%	6.1%	3.9%	7.5%	0.7%	55.2%	61.3%
Morocco	49.7%	2.7%	5.4%	0.9%	4.8%	0.7%	33.9%	39.3%
El Salvador	6.8%	0.6%	43.4%	2.3%	1.5%	0.9%	42.5%	85.9%
Costa Rica	10.6%	1.2%	41.9%	2.9%	6.2%	0.6%	33.5%	75.3%
Madagascar	26.0%	1.3%	5.9%	0.9%	13.2%	1.0%	49.8%	55.6%
Cameroon	44.9%	2.2%	4.8%	0.9%	9.7%	0.4%	33.9%	38.7%
Uruguay	11.4%	1.4%	7.9%	0.6%	10.0%	0.5%	57.0%	64.9%
DominicanRepublic	7.3%	1.0%	46.5%	4.3%	1.3%	0.8%	37.6%	84.1%
Cyprus	53.3%	9.0%	1.6%	4.2%	1.4%	0.2%	28.9%	30.5%
Guatemala	5.9%	0.4%	39.0%	4.3%	1.3%	0.6%	44.9%	83.9%
Jamaica	6.4%	2.9%	38.5%	4.3%	2.2%	1.7%	41.9%	80.5%
Nicaragua	5.9%	0.7%	23.8%	2.1%	6.3%	0.6%	57.9%	81.7%

Source:IMF/Author calculations
USD indicates the total USD when the other portion is approximated by the USD*

Table 9: *Optimal Reserve Levels based on the cost-benefit approach*

Country	Optimal Cost-Benefit Reserve Level (In USD Billions)	Actual Reserves (In USD Billions)	Excess (In USD Billions)
Sri Lanka	3.14	5.2	2.06
Tunisia	6.26	6.7	0.44
Kenya	1.37	9.8	8.43
Morocco	5.69	25.5	19.81
El Salvador	1.67	3.3	1.63
Costa Rica	1.42	7.96	6.54
Madagascar	2.38	6.9	4.52
Cameroon	N/A	2.7	2.7
Uruguay	4.31	13.8	9.49
DominicanRepublic	6.32	5.3	-1.02
Cyprus	N/A	0.94	0.94
Guatemala	4.18	8.8	4.62
Jamaica	1.11	2.8	1.69
Nicaragua	1.47	2.5	1.03

Source:Author calculations based on IMF/Worldbank and respective central bank data
Note:Optimal cost-benefit reserve level is sensitive to the λ. Also,N/A indicates when the γ is not sufficiently greater and λ is closer to the zero

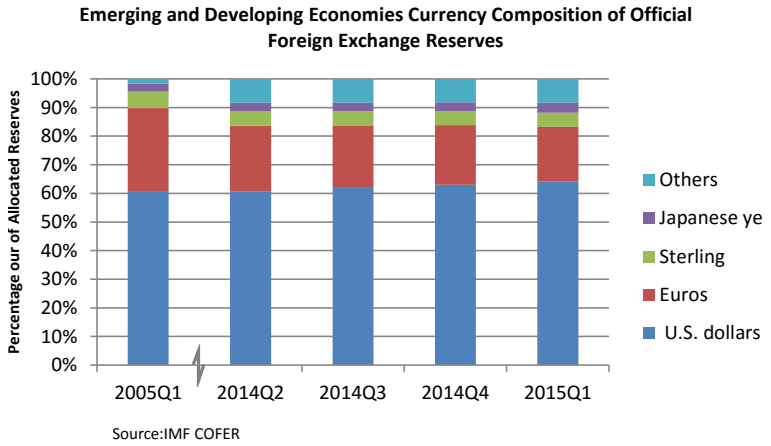
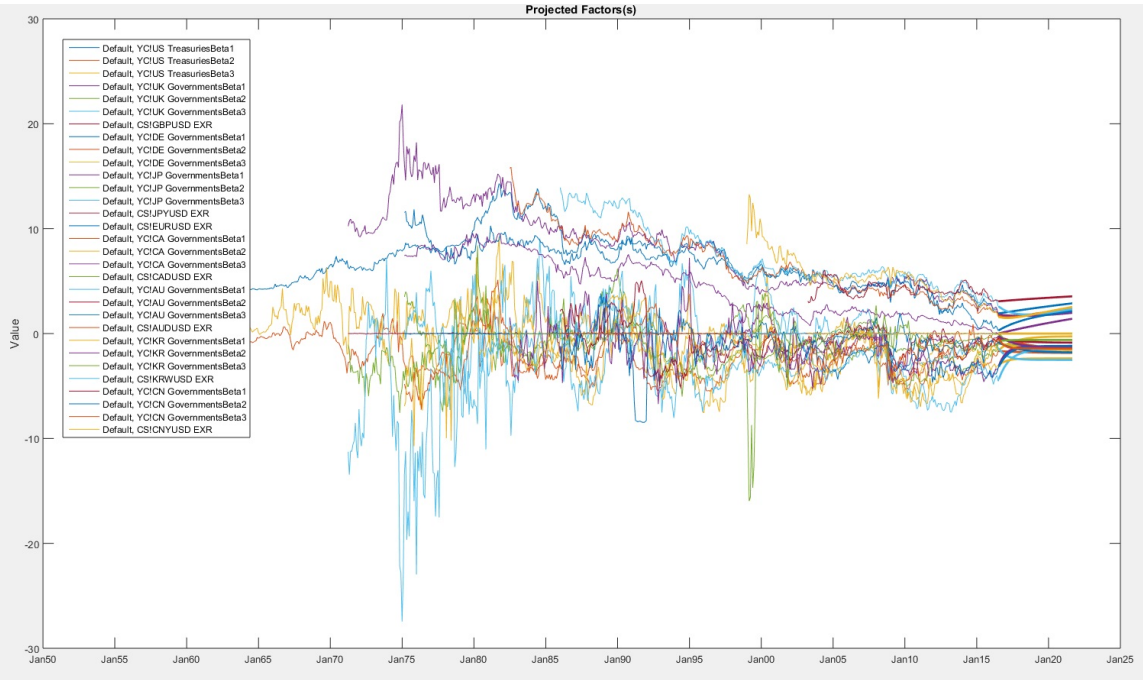


Figure 7: *Actual reserve currency composition: A Comparison 2005 and latest available four quarters*

C. List of Tables and Figures of the Section 2



Source:Author calculations based on factor-based model

Figure 8: *All the empirical and predicted factors using the autoregressive model in the Section 2.2*

D. List of Tables and Figures of the Section 3

Table 10: *Asset Selection*

Asset	Currency
US Govermnt 3 Months	USD
US Govermnt 1 Year	USD
US Govermnt 2 Year	USD
US Govermnt 3 Year	USD
US Govermnt 5 Year	USD
US Govermnt 10 Year	USD
German Government 0-3 Year	EUR
Japanese Government 1 year	JPY
U.K Government 1 year	GBP
Australian Government 0-3Year	AUD
Canadian Government 0-3 year	CAD

Note:Asset Duration is in accordance to the respective Merrill Lynch Global Bond Indices