# Optimal Credit Allocations*

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**Abstract**

This paper examines how commercial banks should allocate their deposits among three different agents: households, firms, and the government to achieve socially optimal allocations. The paper finds that the main driving force of the allocation of loans among three agents are the discount factor, which represents the interest rate, and the risk factor, which is associated with each agent. The discount factor does not exert an important influence on optimal loan allocations when risk is high but becomes highly influential in the presence of low risk. Quantitatively, when the risk of households increases, optimal loan to households converges to zero. On the other hand, when the risk of the government decreases, the optimal loan to the government reaches its upper bound, which is 55% of total loans. Also, a standard calibration of the model reveals that 60% of the total loan should allocate equally between households and firms and the rest should allocate to the government.

*Keywords:* Risk and discount factors, Real shocks, Social optimal allocation

*JEL:* E32, E43

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# 1 Introduction

A commercial bank is a financial institution that is authorized by law to receive money from businesses and individuals and lend money to other individuals and businesses. That is, the primary function of commercial banks is accepting deposits and granting loans. Commercial banks are open to the public and serve individuals, institutions, and businesses. Charging high rates of interest from borrowers while paying a low rates of interest to depositors, the banks' lion share of profits come from this difference. In this paper, I assume that commercial banks take deposits only from patient households as a liability and allocate those liabilities to impatient households, firms, and the government. Therefore, there is an importance to examine how commercial banks should allocate to the aforesaid sectors so that this distribution gives the highest benefit to the society. Interestingly, the results of this research roughly overlaps with the Federal Reserve Board data of 2014 where commercial banks in the United States invest 27.6% in securities, and total loans (both households and firms) are 75.4% as a percentage of deposits. Since studies that directly discuss banks using Dynamic Stochastic General Equilibrium (DSGE) models are scarce in the field of academia, this paper would be an attempt to provide new insights to this discussion.

Kollintzas, Konastantakoloulou, and Tsionas (2011) illustrate domestic credit is pro-cyclical over the business cycle for nine OECD countries. Deposits and long-term interest rates tend to lead the cycle, while credit and short term interest rates lag. An important question is what kind of credit would it be? And are these credit socially optimal? Because, if commercial banks allocate more credit to the household, price level will increase since the higher demand. On the other hand, if commercial banks allocate more credit to the firm, it may create a deflationary situation in the economy due to higher supply. Therefore, allocation of loans is important for macroeconomic variables. Gerali, Neri, Sessa,and Signoretti (2010) is motivational to this study as they have inserted the banking system into classical DSGE model and studied the role of credit supply factors in business cycle fluctuations. They assume banks issue loans only for households and firms and obtain funds via deposits and accumulate capital. However, they have not addressed issues regarding the amount of loan allocation to each agent. In addition, Iacoviello (2014) and Monacelli (2006) also proposed a DSGE model with credit friction in banking sector, which treats as intermediaries between savers and borrowers. However, they have not addressed the amount of loan allocations to each agent, so that it shares optimal allocation of resources of the bank among all agents in the economy.

Therefore, the following research questions are addressed in this paper; (i) What is the socially optimal allocation of loans among three agents: impatient households, firms, and the government by commercial banks? (ii) What is the difference of loan allocation among three agents between socially optimal allocations and the private sector allocations? (iii) What are the characters of each agent that commercial banks look for, and how loan allocation varies per those factors among three agents? (iv) How does loans respond to technology shocks and government expenditure shocks?

This paper suggests a DSGE model with flexible prices having four types of agents: commercial banks, firms, government, patient households and impatient households. According to the results, there is no difference between optimal policy solutions and the private market solutions of allocation of loans among three agents by the commercial banks. Quantitatively, around 62% of the total loans should equally allocate to impatient households and firms and rest should allocate to the government. However, the allocation of loans to each agent changes when the subjective discount factor and the risk factor of each of the agents change.

The remainder of the paper proceeds as follows. Section 2 outlines the model and equilibrium conditions is presented in Section 3. Section 4 describes the Ramsey planer problem. Section 5 describes the calibration of the model . The main quantitative results are presented in Section 6. Finally, Section 7 concludes. Appendix A to F contains the solution of the Ramsey model and private sector equilibrium model.

# 2 The Model

The economy is populated by the government, a representative firm, a representative commercial bank and two types of households: the patient household (saver) and the impatient household (borrower). Households work, consume and deposit resources into a bank, but only the patient household deposit at commercial banks. Subject to the balance sheet constraints, the commercial bank, then, allocates deposited money among three different agents: impatient households, firms, and the government. The commercial bank is the only financial intermediary that handles all financial transactions in the economy. Firms produce final goods and borrow from bank. Figure 1 summarizes the model setup of the paper.

## 2.1 Households

The Preferences of the representative household are defined over consumption ($C_t$) and labor supply ($N_t$). The patient household maximize their present discounted value of lifetime utility by choosing $\left\{C_{p,t}, D_{t+1}, N_{p,t}\right\}$, where as the impatient household chooses $\left\{C_{i,t}, L_{i,t+1}, N_{i,t}\right\}$ to maximize their present discounted life time utility.

### 2.1.1 Patient Households

Patient households deposit (save) within commercial banks and pay a lump-sum tax in each period while receiving labor income and profits from banks as sources of wealth. The preferences of the

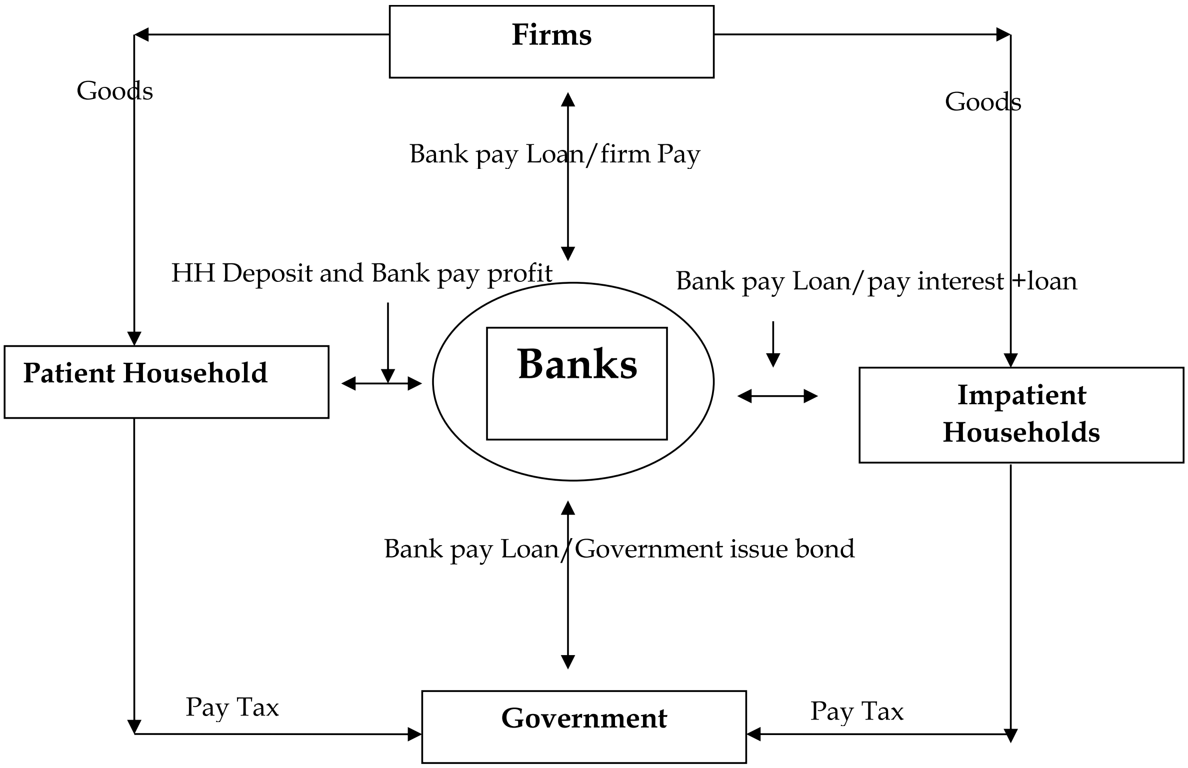


Figure 1: Graphical view of the overall model

patient household are defined over consumption good $(C_{p,t})$ and labor $(N_{p,t})$. The period utility function for the patient household is written as the time separable utility function $U^p\left(C_{p,t}, N_{p,t}\right) = \left[\ln C_{p,t} - \theta_p \frac{N_{p,t}^{1+\chi}}{1+\chi}\right]$ which has the following properties: $\frac{\partial U}{\partial C} > 0$, $\frac{\partial^2 U}{\partial C^2} < 0$, $\frac{\partial U}{\partial N} < 0$ and $\frac{\partial^2 U}{\partial N^2} < 0$. Then the household maximizes their expected present discounted lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} (\beta_p)^t U^p \left[C_{p,t}, N_{p,t}\right]$$

subject to the sequence of period budget constraints:

$$C_{p,t} + D_{t+1} + I_t = W_t N_{p,t} + R_t K_t + \left(1 + r_{p,t-1}\right) D_t + \Pi_t - T_{p,t}$$

and capital accumulation condition given by:

$$K_{t+1} = I_t + (1-\delta) K_t$$

where $\beta_p \in (0,1)$ is the subjective discount factor of the patient household, $E_t$ denote the expectation operator conditional on information available at the beginning of the period $t$, $\theta_p$ denotes dis-utility of labor, $\chi$ is the elasticity of labor supply with respect to the wage. When the wage is high, the labor supply is more responsive to the fluctuations in the wage, $\Pi_t = \left(\pi_t^b\right)$ is defined as profit of banks, $T_{p,t}$ is lump sum tax of the patient household, $r_{p,t}$ is the interest rate for the deposit, $D_t$ is the supply of deposit by the patient household.

The optimal choices of consumption, labor supply, capital and deposit holdings leads to the following conditions respectively:[1]

$$\theta_p N_{p,t}^{\chi} C_{p,t} = W_t \tag{1}$$

$$\frac{1}{C_{p,t}} = \beta_p E_t \left[ \frac{1}{C_{p,t+1}} \left( R_{t+1} + 1 - \delta \right) \right] \tag{2}$$

$$\frac{1}{C_{p,t}} = \beta_p E_t \left[ \frac{1}{C_{p,t+1}} \left( 1 + r_{p,t} \right) \right] \tag{3}$$

where equation (1) is the labor supply condition, equation (2) is the capital supply condition and the final equation (3) is the consumption Euler condition.

### 2.1.2 Impatient Households

The impatient household also has the same utility function. However, they borrow $\left(L_{i,t}\right)$ from the commercial bank at an interest rate $\left(r_{i,t-1}\right)$. Also firms belong to the impatient households and received profit. The preferences of the representative impatient household is defined over consumption $\left(C_{i,t}\right)$ and the labor supply $\left(N_{i,t}\right)$. Therefore, the impatient household maximizes their expected discounted lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \left(\beta_I\right)^t U^I \left[ C_{i,t}, N_{i,t} \right]$$

subject to the sequence of period budget constraints:

$$C_{i,t} + \left(1 + r_{i,t-1}\right) L_{i,t} = W_t N_{i,t} + L_{i,t+1} + \Pi_t - T_{i,t}$$

where $U^I \left(C_{i,t}, N_{i,t}\right) = \left[ \ln C_{i,t} - \theta_I \frac{N_{i,t}^{1+\chi}}{1+\chi} \right]$ which also satisfy the above properties of utility function, $\beta_I \in (0,1)$ is the subjective discount factor of the impatient household $\left(\beta_p > \beta_I\right)$. $\Pi_t = \left(\pi_t^f\right)$ is defined as the profit from the firm.The Impatient household also receives labor income, profits from firm and pay lump sum tax. Impatient households optimally choose consumption, labor, and loans for each period to maximize the present discount lifetime utility and then the first order conditions

[1]Mathematical derivations are provided in the Appendix A

yeild the following conditions[2]:

$$\theta_I N_{i,t}^{\chi} C_{i,t} = W_t \tag{4}$$

$$\frac{1}{C_{i,t}} = \beta_I E_t \left[ \frac{1}{C_{i,t+1}} \left(1 + r_{i,t}\right) \right] \tag{5}$$

where equations (4) and (5) are, respectively, the labor supply condition for the impatient household and the consumption Euler condition.

## 2.2 Firms

The representative firm hires labor at the real wage rate $W_t$, and rents capital at the rate $R_t$, take the loan $\left(L_{f,t}\right)$ at the rate $r_{f,t-1}$ to produce a final good $(Y_t)$ usage of the following constant returns to scale production technology:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \; A_t > 0 \text{ and } \alpha \in (0, 1)$$

where $N_t = \left(N_{p,t} + N_{i,t}\right)$, $A_t$ is the total factor productivity. Firms' problem can be written as the following maximization problem:

$$E_0 \sum_{t=0}^{\infty} M_t \left( A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - R_t K_t + L_{f,t+1} - \left(1 + r_{f,t-1}\right) L_{f,t} \right) \tag{6}$$

where $M_t$ is the stochastic discount factor which discount future cash flows. Define cash flows in terms of current consumption $(t = 0)$. Then the firm stochastic discount factor can be written as the following form since consumers value future dividends flows in this format:

$$M_t = \beta_f^t \frac{U^{I'}\left(C_{i,t}\right)}{U^{I'}\left(C_{i,0}\right)}$$

where $U^{I'}\left(C_{i,t}\right)$ is the additional unit of utility at time $t$ generated by one unit of dividends, $\beta_f^t$ is the discount factor for the firm, $U^{I'}\left(C_{i,0}\right)$ is the consumption equivalent value of the future utils. The representative firm maximizes the present discounted value of net revenue by choosing $\left\{K_t, N_t, L_{f,t+1}\right\}$. Then, first order conditions give factor prices equal to their marginal products[3]:

$$(1 - \alpha) N_t^{-\alpha} A_t K_t^{\alpha} = W_t \tag{7}$$

$$\alpha N_t^{1-\alpha} A_t K_t^{\alpha-1} = R_t \tag{8}$$

[2]Mathematical derivations are provided in the Appendix B

[3]Mathematical derivations are provided in the Appendix C

$$\frac{1}{C_{i,t}} = \beta_f E_t \left[ \frac{1}{C_{i,t+1}} \left(1 + r_{f,t}\right) \right] \tag{9}$$

where equations (7) and (8) are, respectively, marginal product of labor and marginal product of capital.

## 2.3 Government

The government activities are divided into two parts: fiscal policy which is conducted by the treasury department, and the monetary policy which is conducted by the central bank. However, this study is not focusd on the monetary policy. The government chooses its spending exogenously and finance it by lump-sum taxes from two different households and by taking loan from the commercial banks. Then the governments budget constraint can be written as:

$$G_t + r_{g,t-1} L_{g,t} = T_t + L_{g,t+1} - L_{g,t} \tag{10}$$

where $G_t$ is government spending, $r_{g,t-1} L_{g,t}$ is the interest payment on the total government debt, $T_t$ is the net taxes, $\left(L_{g,t+1} - L_{g,t}\right)$ is the purchase of new loans.

## 2.4 Commercial Banks

Commercial banks collect the deposit from patient households and makes loans to other agents in the economy. Commercial banks, therefore, are the only financial intermediary that provide loans to impatient households, firms and to the government, with which patient households can save their money. In addition, the banking sector is characterized by a continuum of banks with perfect competition. Many papers have discussed the banking sector with the DSGE model in different ways. Gerali et al. (2010), for instance, have introduced two types of banks; wholesale branches, and retail branches and introduced monopolistic competition at the banking retail level. Commercial banks, in general, use the deposits and bank equity as the factors to determine the loans which they finance. Loans issued by wholesale branches are mainly decided on wholesale deposits and bank capital, where, in contrast, retail branches use loans received by wholesale branches and issue to the impatient household and firms. By doing so, Banks bare a quadratic adjustment cost for changing the rates they charge on loan. Falagiarda et al. (2013) assess the perfect competition banking sector with the DSGE model. In contrast to the Gerali (2010) study, Falagiarda (2013) have showed two distinct differences. First, the commercial bank uses equity, deposits and borrowing from central bank as a source of funds (liability) for issuing loan to households and buying government bond. Secondly, the bank bares two different quadratic costs. In terms of equity, these costs are the the costs they bare when they move away from a leverage ratio and are bared from issuing loans to households. In addition, Gertler and Kiyotaki (2010), Goodhart et al. (2009), Dib (2010), have introduced banking sector in to the DSGE model. All above studies, maximize profit function of the bank subjected to the balance sheet identity, yet, Iacoviello (2014) have considered bank as a different agent that maximizes lifetime consumption in log format subjected to the budget constraints. He also introduces a quadratic portfolio loan adjustment cost.

This paper, to that end, have introduced three different quadratic cost functions on loans issued for impatient household, firms and government. Cost of each loan varies as the cost associated with risk and other transaction cost. Figure 2 represents the basic structure of the representative bank

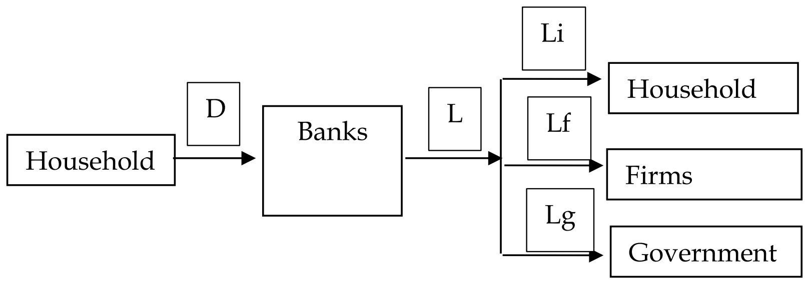


Figure 2: Graphical view of flow of banks asset and liability

Assume there is no equity at the beginning of each period. Therefore, the balance sheet identity can be written in the following format:

$$L_{i,t} + L_{f,t} + L_{g,t} = L_t$$

where $L_t$ is the total loan, $L_{i,t}$ is the loan to impatient households, $L_{f,t}$ is the loan to firms and $L_{g,t}$ is the loan to government. Also at the same time the balance sheet identity can be written as:

$$D_{t+1} = L_{i,t+1} + L_{f,t+1} + L_{g,t+1} = L_{t+1}$$

where $D_{t+1}$ is the patient household deposit. Let us assume interest rates for the loan of impatient households, firms, and the government are, respectively, $r_{i,t}$, $r_{f,t}$ and $r_{g,t}$. Also, the interest rate on deposits is $r_{p,t}$. By changing discount factor, we can adjust interest rates on loans and deposits. So, the representative bank solves the following problem by choosing $\left\{L_{i,t+1}, L_{f,t+1}, L_{g,t+1}, D_{t+1}\right\}$ to maximize the expected discounted profit $E_0 \sum\limits_{t=0}^{\infty} B_t \pi_t$.

Then bank problem can be written as:

$$E_0 \sum_{t=0}^{\infty} B_t \Big( D_{t+1} + \left(1 + r_{f,t-1}\right) L_{f,t} + \left(1 + r_{g,t-1}\right) L_{g,t} + \left(1 + r_{i,t-1}\right) L_{i,t} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} - \left(1 + r_{p,t-1}\right) D_t - \frac{\phi_f}{2} L^2_{f,t+1} - \frac{\phi_g}{2} L^2_{g,t+1} - \frac{\phi_i}{2} L^2_{i,t+1} \Big) \tag{11}$$

subject to the balance sheet identity:

$$D_{t+1} = L_{f,t+1} + L_{g,t+1} + L_{i,t+1} \tag{12}$$

where $B_t = \beta_B^t \frac{U^{P'}(C_{p,t})}{U^{P'}(C_{p,0})}$ is the stochastic discount factor of the bank since banks belong to the patient households. $\beta_B$ is the subjective discount factor of the bank and $\phi_f$, $\phi_g$, $\phi_i$ are respectively, risk associated with firm, the government, and the impatient household. In this profit function, the first term is for the deposit by the patient household and then next three terms are for interest and loan payment for the last period of the loan. Then first four terms are considered as a source of funds. The next three terms are for the allocation of the loan for households, firms, and the government. Finally, the last three terms are for the cost of bearing by the commercial bank to allocate loans among three agents. This cost associated with administrative costs and risk of loans.

The first order conditions with respect to choice variables $\{D_{t+1}, L_{f,t+1}, L_{g,t+1}, L_{i,t+1}\}$ are as follows[4]:

$$B_t(1+\lambda_t) - E_t\left[B_{t+1}\left(1+r_{p,t}\right)\right] = 0 \;\Rightarrow\; B_t(1+\lambda_t) = E_t\left[B_{t+1}\left(1+r_{p,t}\right)\right] \tag{13}$$

$$\phi_f \frac{1}{C_{p,t}} L_{f,t+1} = \beta_B E_t\left[\frac{1}{C_{p,t+1}}\left(r_{f,t} - r_{p,t}\right)\right] \tag{14}$$

$$\phi_g \frac{1}{C_{p,t}} L_{g,t+1} = \beta_B E_t\left[\frac{1}{C_{p,t+1}}\left(r_{g,t} - r_{p,t}\right)\right] \tag{15}$$

$$\phi_i \frac{1}{C_{p,t}} L_{i,t+1} = \beta_B E_t\left[\frac{1}{C_{p,t+1}}\left(r_{i,t} - r_{p,t}\right)\right] \tag{16}$$

Simplifying equation $(13)$ by substituting stochastic discount factor, gives the Lagrange multiplier in terms of deposit rate and the subjective discount factor of the bank. Then equation $(14)$,$(15)$ and $(16)$ gives loans supply relation to firms, government and impatient households respectively.

## 2.5 Exogenous Processes

There are two exogenous processes that are considered in this analysis: productivity and government expenditure.

### 2.5.1 Productivity

Total factor productivity ($A_t$) follows first order autoregressive (AR(1)) process in logs terms:

$$\ln A_t = (1-\rho_a)\ln A + \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{17}$$

where $A > 0$ is steady-state level of the total factor productivity process, and $0 < \rho_a < 1$ is the AR(1) persistence parameter. $\varepsilon_{a,t}$ is a random shock to the total factor productivity, and has a normal distribution, that is $\varepsilon_{a,t} \sim N\left(0, \sigma_a^2\right)$.

[4]Mathematical derivations are provided in the Appendix D

### 2.5.2 Government Expenditure

In addition, government spending is defined to be a stochastic variable and follows a first order auto regressive process:

$$\ln g_t = \left(1 - \rho_g\right) \ln(\omega g) + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \tag{18}$$

where $g > 0$ is steady-state level of the government expenditure process, $0 < \rho_g < 1$, is the AR(1) persistence parameter, and $\varepsilon_{g,t}$ is a random shock to the government expenditure, and has a normal distribution; that is, $\varepsilon_{g,t} \sim N\left(0, \sigma_g^2\right)$.

## 2.6 Private Market equilibrium

A competitive equilibrium is a set of planes which combine with set of prices $\left\{w_t, r_{f,t}, r_{i,t}, r_{g,t}, R_t\right\}$ and allocations $\left\{C_{p,t}, N_{p,t}, D_{t+1}, K_{t+1}, C_{i,t}, N_{i,t}, L_{f,t+1}, L_{i,t+1}, L_{g,t+1}\right\}$ such that all agents patient households, impatient households and firms equilibrium (optimality) conditions hold which are derived from both patient households and impatient households expected lifetime utility maximization by considering prices as given and the firm profit maximization taking as given wage rate and capital rent rate. In addition, all market should clear. That is the labor market clear since the firms hires all the labor which supply by the both types of households, bond market clears since patient households hold all bond which are issued by the government, and capital market clears. Further, government budget constraint and aggregate resource constraint are satisfied. Here resource constraint[5] can be written as:

$$Y_t = C_t + I_t + G_t + \frac{\phi_f}{2} L_{f,t}^2 + \frac{\phi_g}{2} L_{g,t}^2 + \frac{\phi_i}{2} L_{i,t}^2 \tag{19}$$

# 3 Private Market equilibrium conditions and the Market clearing conditions

This section combines the first order conditions of the impatient household, the patient household and firms problem with the market clearing conditions to yield the equilibrium conditions:

$$\frac{1}{C_{p,t}} = \beta_p E_t \left[\frac{1}{C_{p,t+1}} \left(R_{t+1} + 1 - \delta\right)\right] \tag{20}$$

$$\frac{1}{C_{p,t}} = \beta_p E_t \left[\frac{1}{C_{p,t+1}} \left(1 + r_{p,t}\right)\right] \tag{21}$$

$$\theta_p N_{p,t}^{\chi} C_{p,t} = W_t \tag{22}$$

[5] Mathematical derivations are provided in the Appendix D

$$\frac{1}{C_{i,t}} = \beta_I E_t \left[ \frac{1}{C_{i,t+1}} \left(1 + r_{i,t}\right) \right] \tag{23}$$

$$\theta_I N_{i,t}^{\chi} C_{i,t} = W_t \tag{24}$$

$$C_{i,t} + \left(1 + r_{i,t-1}\right) L_{i,t} = W_t N_{i,t} + L_{i,t+1} + T_{i,t} \tag{25}$$

$$(1-\alpha) N_t^{-\alpha} A_t K_t^{\alpha} = W_t \tag{26}$$

$$\alpha N_t^{1-\alpha} A_t K_t^{\alpha-1} = R_t \tag{27}$$

$$\frac{1}{C_{i,t}} = \beta_f E_t \left[ \frac{1}{C_{i,t+1}} \left(1 + r_{f,t}\right) \right] \tag{28}$$

$$G_t + r_{g,t-1} L_{g,t} = T_t + L_{g,t+1} - L_{g,t} \tag{29}$$

$$\phi_f \frac{1}{C_{p,t}} L_{f,t+1} = \beta_B E_t \left[ \frac{1}{C_{p,t+1}} \left(r_{f,t} - r_{p,t}\right) \right] \tag{30}$$

$$\phi_g \frac{1}{C_{p,t}} L_{g,t+1} = \beta_B E_t \left[ \frac{1}{C_{p,t+1}} \left(r_{g,t} - r_{p,t}\right) \right] \tag{31}$$

$$\phi_i \frac{1}{C_{p,t}} L_{i,t+1} = \beta_B E_t \left[ \frac{1}{C_{p,t+1}} \left(r_{i,t} - r_{p,t}\right) \right] \tag{32}$$

$$Y_t = A K_t^{\alpha} \left(N_{i,t} + N_{p,t}\right)^{1-\alpha} \tag{33}$$

$$K_{t+1} = I_t + (1-\delta) K_t \tag{34}$$

$$Y_t = C_t + I_t + G_t + \frac{\phi_f}{2} L_{f,t}^2 + \frac{\phi_g}{2} L_{g,t}^2 + \frac{\phi_i}{2} L_{i,t}^2 \tag{35}$$

$$T_t = T_{p,t} + T_{i,t} \tag{36}$$

$$C_t = C_{i,t} + C_{p,t} \tag{37}$$

$$N_t = N_{i,t} + N_{p,t} \tag{38}$$

and close the model with two exogenous stochastic processes for government expenditure and technology:

$$\ln A_t = \rho_a \ln A_{t-1} + v \tag{39}$$

$$\ln g_t = \left(1 - \rho_g\right) \ln(\varpi g) + \rho_g \ln g_{t-1} + u. \tag{40}$$

Then, using above equilibrium condition we can find the private market solutions which we compare with the following Ramsey planner solutions.

# 4 The Ramsey Problem

In the present time, Ramsey planer problem is popular method to find the optimal monetary policy and optimal fiscal policy. Recent studies by Khan et al (2003), Schmitt-Grohe and Uribe (2004), and Monacelli (2006) have used Ramsey type approaches in DSGE model to analysis the optimal policy. This section sets up the optimal policy that maximizes the welfare function subject to private market equilibrium conditions and the resource constraint in the economy. Ramsey planer maximizes the weighted average utility function since there are two types of households patient households and impatient households . Define $\omega$ and $(1-\omega)$ as the weights assigned to patient households and impatient households respectively. Then Ramsey planer maximizes the following weighted utility function: $W = \omega \sum_{t=0}^{\infty} \beta_p^t U^p \left(C_{p,t}, N_{p,t}\right) + (1-\omega) \sum_{t=0}^{\infty} \beta_I^t U^I \left(C_{i,t}, N_{i,t}\right)$ for a given stochastic process $\{A_t, G_t\}_{t=0}^{\infty}$. Social welfare function can be defined in different ways. However, in this study, I have considered it as in the above additive format. Then, if $\omega = 0.5$ both households in the social welfare function treat equally In the optimal policy, there are two types of variables, co-state variables and control variables, chosen to maximize the problem. The co-state variables are the Lagrange multipliers on each constraint. Define the Lagrange multiplier by $\Delta^t \lambda_{k,t}$, where $\Delta = \beta_I^{\omega} \beta_p^{1-\omega}$ and $\left\{\lambda_{k,t}\right\}$ are $\left\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}, \lambda_{11,t}, \lambda_{12,t}, \lambda_{13,t}\right\}_{t=0}^{\infty}$, for number of constraints. Control variables are all the choice variables in private market solution $\left\{C_{p,t}, N_{p,t}, D_{t+1}, K_{t+1}, C_{i,t}, N_{i,t}, L_{f,t+1}, L_{i,t+1}, L_{g,t+1}\right\}$ and price vector $\left\{w_t, r_{f,t}, r_{i,t}, r_{g,t}, R_t\right\}$. Then Ramsey planers' maximization problem can be written as:

$$W = \omega \sum_{t=0}^{\infty} \beta_p^t U^p \left(C_{p,t}, N_{p,t}\right) + (1-\omega) \sum_{t=0}^{\infty} \beta_I^t U^I \left(C_{i,t}, N_{i,t}\right)$$

subject to the constraints (1), (2), (3), (4), (5), (7), (8), (9), (10), (14), (15), (16), (19) and two exogenous process.

# 5 Calibration

This section analyzes the quantitative predictions of the model by studying the results of numerical simulation of an economy. Most parameter values in this paper are based on Gerali et al. (2010), Iacoviello (2014) and Monacelli (2006). Time is measured in quarterly. Subjective discount factor $(\beta_p)$ of patient households set as 0.99 so that the annual interest rate for deposit is equal to 4 percent. Monacelli (2006) and Faia (2007) also set patient households (saver) discount factor as 0.99. However, Gerali et al. (2010) set the patient household discount factor at 0.9943 in order to set the deposit rate slightly above 2 percent. Discount factors $(\beta_I)$ and $(\beta_f)$ of impatient household and firms respectively, set at 0.95 to satisfy Iacoviello (2014), and Gerali et al. (2010) values. This study sets dis-utility of labor $(\theta_p, \theta_I \geq 0)$ (which is different from study to study) as 5.25 and 5.25 respectively. If the parameter $\theta$ is different in the utility functions of households, then there will be two utility function for households. The higher $\theta_p, \theta_I$ represents the lower average labor supply as the household are resistant to excess work. $\chi \geq 0$ is the elasticity of labor supply with respect to the wage. When $\chi$ is high, the labor supply is more responsive to fluctuation in the wage. The wage is high when the economy is productive. In this paper, I set it equal to 0.35. Then, $\alpha \geq 0$ is the elasticity of capital and $1 - \alpha$ is the elasticity of labor. The production function has constant returns to scale. The study sets $\alpha = 0.30$ and the depreciation of capital $\delta = 0.025$ as these values are commonly used in the literature. Welfare function is introduced by giving equal weight to impatient households and patient household. That is, set $\omega = 0.50$ as Iacoviello (2014) suggests in his paper. Then it implies the equal impact to social welfare function from both households. However, paper have showed effect of $\omega$ to the loans allocation of each agents. The fact that the risk factor of the government is five times lower than the other two agents' households and firms was instrumental to set the risk factor values unique to this study. However, usually we assume that there is no risk on government. Then, all the parameter values of model listed in the table 1. That is it shows the parameters for the bench mark model. Change of any parameter given would result a change in steady state values and the ratio of loans. That is steady state values and ratio of loan change accordingly. Similarly, these parameter values are common in OECD countries and in the USA. This paper provides evidence how results could change when discount factor and risk factors of each agents are changed.

Table 1: Calibrated parameters for the model

| Symbols | Value | Description |
|---|---|---|
| $\beta_p$ | 0.99 | Subjective discount factor for the patient household |
| $\beta_I$ | 0.96 | Subjective discount factor for the impatient household |
| $\beta_f$ | 0.96 | Subjective discount factor for firms |
| $\beta_B$ | 0.99 | Subjective discount factor for banks |
| $\alpha$ | 0.30 | Capital share of production |
| $\chi$ | 0.35 | Elasticity of labor supply with respect to wage |
| $\theta_p$ | 5.25 | Disutility of labor by patient household |
| $\theta_I$ | 5.25 | Disutility of labor by impatient household |
| $\phi_I$ | 0.015 | Risk factor of impatient household on loan |
| $\phi_g$ | 0.003 | Risk factor of government on loan |
| $\phi_f$ | 0.015 | Risk factor of firm on loan |
| $\omega$ | 0.5 | Ramsey preference weight |
| $\delta$ | 0.025 | Depreciation of capital |
| $\rho_a$ | 0.92 | Serial correlation of technology shocks |
| $\rho_g$ | 0.92 | Serial correlation of government expenditure shocks |
| $y_{ss}$ | 1.5 | Steady state of output |
| $\sigma_z$ | 0.0026 | Standard deviation of the innovation to ln(z) |
| $\sigma_u$ | 0.0018 | Standard deviation of the innovation to government expenditure |

# 6 Results

If we examine the difference between private sector solution and Ramsey planner solution of total loan and the loan ratio ($L$, $\frac{Lf}{L}$, $\frac{Li}{L}$, $\frac{Lg}{L}$) the Table 2 shows the comparison of optimal policy solution and private market solution.

Table 2: Comparisons of loan ratios of each agents: Ramsey vs market solution

| | Ramsey Solution | | Private Sector Solution | |
|---|---|---|---|---|
| | Mean | Std.Dev | Mean | Std.Dev |
| Loan to Impatient HH/Total Loan | 30.23% | 0.0017 | 30.23% | 0.002 |
| Loan to Firm/Total Loan | 30.23% | 0.0017 | 30.23% | 0.002 |
| Loan to Government/Total Loan | 39.53% | 0.0034 | 39.53% | 0.004 |

There is no difference of mean values between optimal policy solution and the private market solution of most important three ratios: loan to impatient households, loan to firm and loan to government. However, there is a difference of standard deviations as it shows these deviations of the Ramsey optimal policy solution are higher than that of private market solutions.
As per the results, loan among agents depend on risk and discount factor (interest rate) associated with each agent. Table 3 shows that when the risk parameter increases from 0.015 to 2, how each loan change accordingly as a percentage to the total loan. In this case it is considered that all agents have same risk factor

Table 3: Comparison of loan ratio to total loan according to the different risk of Ramsey optimal policy problem

| Equal risk among all agents | 0.015 | 0.026 | 0.1 | 1 | 2 |
|---|---|---|---|---|---|
| Loan to Impatient HH/Total Loan | 36.68% | 33.34% | 24.41% | 11.16% | 8.40% |
| Loan to Firm/Total Loan | 36.68% | 33.34% | 24.41% | 11.16% | 8.40% |
| Loan to Government/Total Loan | 26.65% | 33.31% | 51.18% | 77.68% | 83.21% |

Table 3 shows when there is an equally high risk of each agent in the economy, where risk factor is 2 for all agents, banks allocate most of their deposit (83.21%) to the government and remaining (approximately a 17%) is allocated equally among households and firms. However, in the presence of low risk in the economy, where risk factor is 0.015 for all agents, bank allocates the most of funds to impatient households and firms (36.68% each). Further, when the risk is 0.026 for all agents, loans should equally allocate among three agents. On the other hand, when consider the effect of discount factor of impatient households, interest rate or the discount factor play a dominant role in the low risk which is 0.015. Table 4 and Table 5 shows the comparison of the loan ratio to total loan per discount factor in low risk and high risk situations respectively.

In the low risk region (Table 4) , when discount factor increase from 0.90 to 0.989 which is less than patient house hold discount factor, loan allocation to impatient households as a ratio from

Table 4: Comparison of loan ratio to total loan according to discount factor in low risk region ($\phi_i$=0.015, $\phi_g$=0.003, $\phi_f$=0.015) of Ramsey optimal policy problem

| Impatient HH Discount Factor | Loan Impatient HH/Total Loan | Loan Firm/Total Loan | Loan Gov/Total Loan |
|---|---|---|---|
| 0.9 | 58.10% | 18.16% | 23.74% |
| 0.93 | 47.22% | 22.87% | 29.91% |
| 0.95 | 36.86% | 27.36% | 35.77% |
| 0.97 | 22.24% | 33.70% | 44.06% |
| 0.989 | 6.58% | 40.49% | 52.94% |

total loan allocation, decrease from 58.10% to 6.58% because when the discount factor is small then interest rate is high and otherwise.

Table 5: Comparison of loan ratio to total loan according to discount factor in high risk region ($\phi_i$=2, $\phi_g$=0.4, $\phi_f$=2) of Ramsey optimal policy problem

| Impatient HH Discount Factor | Loan Impatient HH/Total Loan | Loan Firm/Total Loan | Loan Gov/Total Loan |
|---|---|---|---|
| 0.9 | 16.55% | 5.17% | 78.28% |
| 0.93 | 11.34% | 5.49% | 83.17% |
| 0.95 | 7.70% | 5.72% | 86.58% |
| 0.97 | 3.93% | 5.95% | 90.12% |
| 0.989 | 1.00% | 6.13% | 92.87% |

When compared to the low-risk situation, the loan allocation for the impatient household is lower in difference, yet still reports a change from 16.55% to a less of 1.00% as shown in Table 5. However, in the high-risk scenario, loan to households decrease about 20% with the increase of the discount factor. This, therefore, suggests that risk factor plays a dominant role with a high-risk economy while the interest rate plays dominant role with a low risk economy.

Figure 3 shows that loan allocation for the impatient households or firm when risk factor is increasing. Here, impatient households risk increases from 0.015 to 1 while firm and government risk remain same as previously at 0.015 and 0.003 respectively.

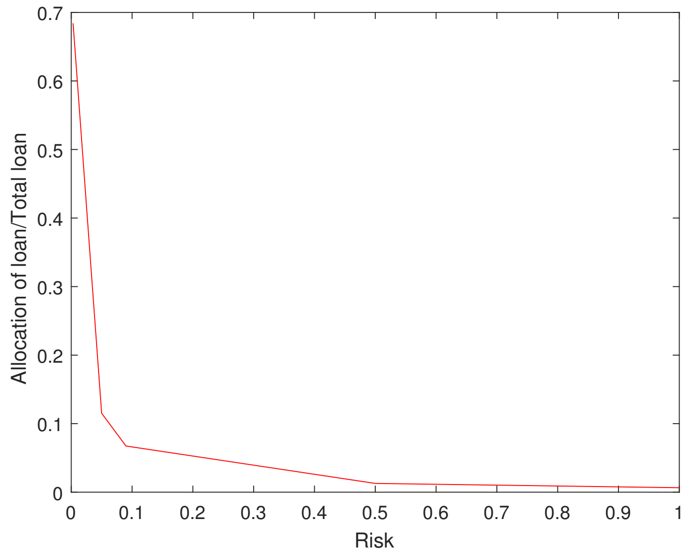


Figure 3: Loan allocation to impatient households for different risk ($\phi_i$=(0.005-1.00), $\phi_g$=0.003, $\phi_f$=0.015) of Ramsey optimal policy problem

When the risk reaches 0.015, more than 30% of loan is allocated to the impatient households. However, when the risk increases to 1, the loan to them is less than 1%. The findings of the research predicts optimal loan allocation could converge to zero, alongside with the rapid increase of the risk factor.

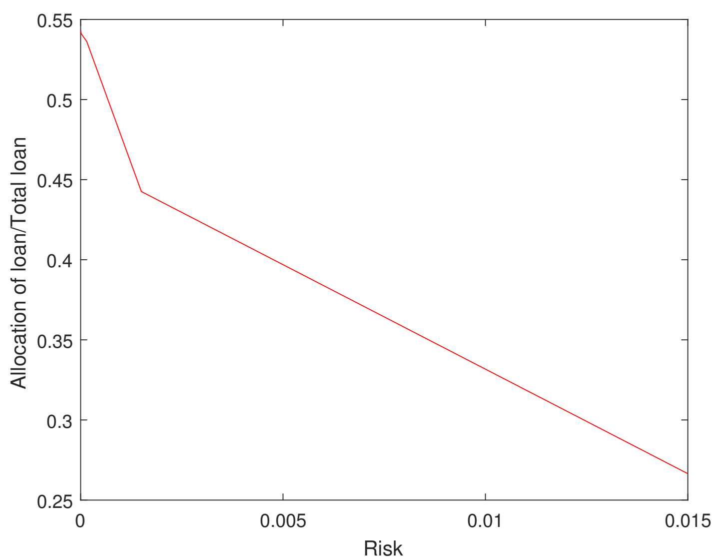


Figure 4: Loan allocation to the government to different risk ($\phi_i$=0.015, $\phi_g$= (0.015-0.000015), $\phi_f$=0.015) of Ramsey optimal policy problem

Figure 4 projects that the gradual decrease of the risk (the risk of government decrease from 0.003 to almost zero risk) results a considerable increase of the loan allocation for the government. The allocation could reach up to its maximum level of 55% from the total loan allocation for that matter. That is, when compared to the other two agents, loan allocation to government has an upper bound of 55%.

As per the above two figures that discuss the behavior of the risk and its consequences on government and other agents, in a loan allocating context, the greater the risk results the minimum allocation of loans to households and firms as it creates a non- performing loan situation for the commercial banks. In contrast, lower risk results a maximum allocation of loans to the government, as it establishes a credible relationship in the loan allocation process with the bank, yet, the maximum limit should not exceed 55% of total loan allocation. This is because the more bank allocates to the government would result in a low interest gain situation.

In this study, I have defined the social welfare function by combining the two utility function assign weights $\omega$ and $(1-\omega)$. The Figure 5 illustrates the impulse response functions to technology shocks for different weights

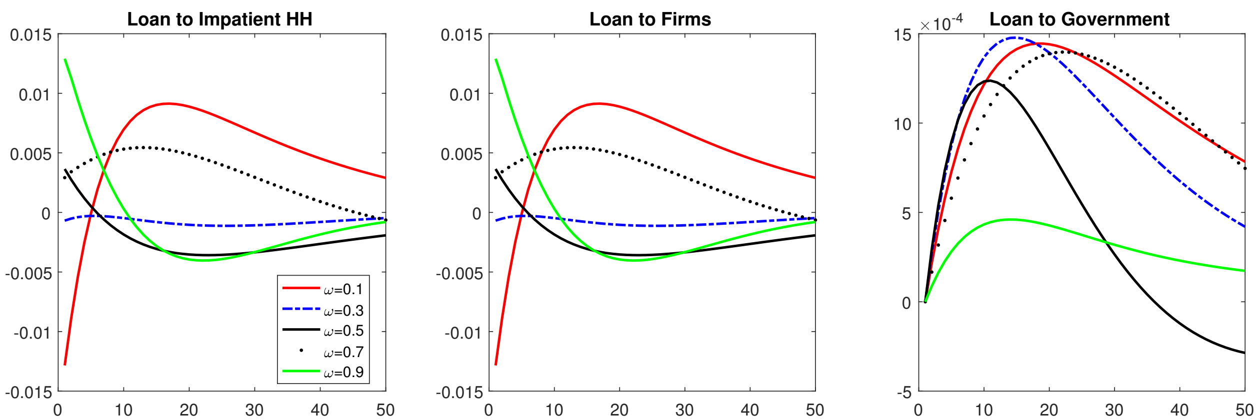


Figure 5: The impulse response functions to technology shocks for different weights.

Impulse response functions of loan to firms and loan to impatient households behave in similar manner for different values of $\omega$. However, it is not the case for loan to the government. Because, weight is directly effect to the impatient households and then firms. However, loan to the government has a similar effect for the both shocks.

Figure 6 illustrates how a technology shock and government expenditure shocks effect for two different models, private market and Ramsey planner. In general, loan to the impatient households and loan to firms behave similarly for both technology and government expenditure shocks. However, in the case of loan to impatient household and firms, government expenditure and technology shocks react oppositely in two markets. However, loan to the government has a similar effect for both shocks.

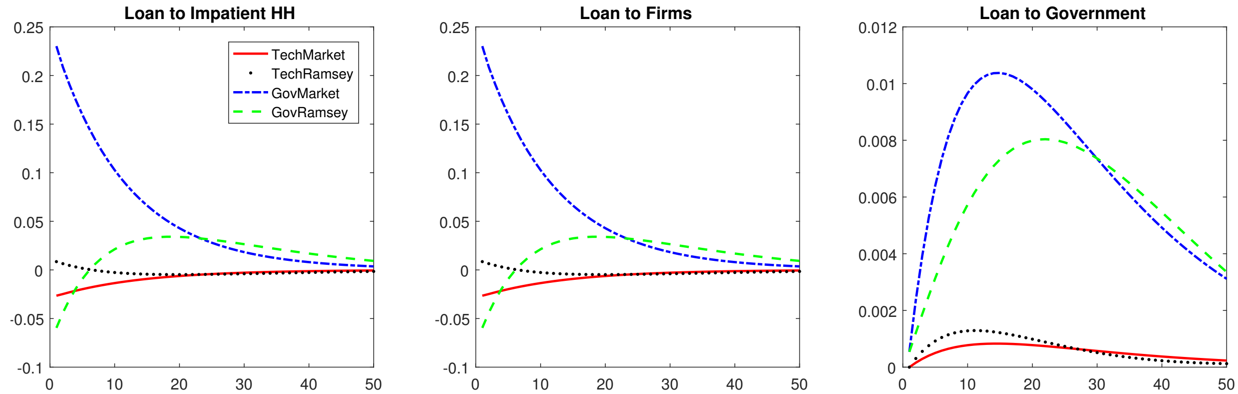


Figure 6: The impulse response functions to technology shocks and government expenditure shocks under Ramsey equilibrium and private market equilibrium for loans to each agent.

The income effect is one possible reason for this behavior. It is clear that, the loan demand decreases along with the increase of income and vice versa. In an increased productivity shock, the labor of impatient household can be decreased while in a government expenditure shock this labor could be increased in the private market solution. In such situation, the wage rate responds in an opposite way. If wage rate is dominant in both cases, income of impatient household will increase when there is a productivity shock while the income will decrease when there is government expenditure shock. Therefore, a negative effect would yield on impatient household loan in a productivity shock and a positive effect of in a government expenditure shock.

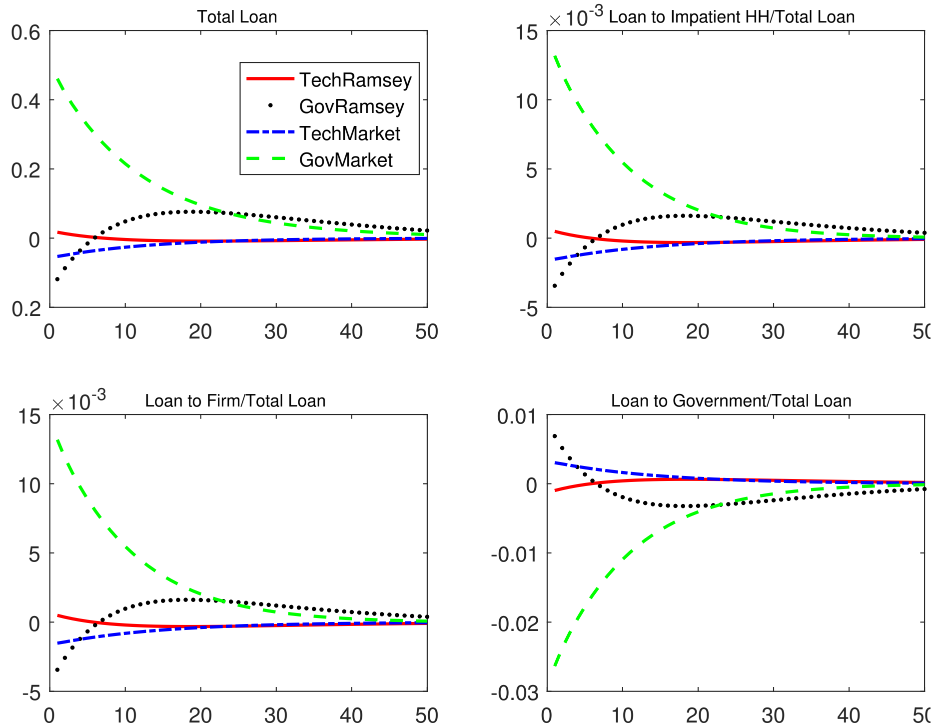


Figure 7: The impulse response functions to technology shocks and government expenditure shocks under Ramsey equilibrium and private market equilibrium.

Figure 7 illustrates the fluctuations of total loan and its ratios related to both households, firms and government. Impulse response function shown in the figure demonstrate the behavior of the total loan and loan ratios related to agents, those experienced two different shocks (Technology and Government expenditure) given to both Ramsey optimal policy solution and the private market solution. Thus, impatient households and firms shows a similar fluctuation to the total loan behavior while the government report its opposite. Table 6 shows the mean and the standard deviation for all the variables that show approximately equal values of both private market problem solution and optimal policy problem solution. Volatility of output and wage are almost equal and high. However, loan allocations among three agents are equal in both cases. But volatility of loan to impatient households is high compared to that of firms and government, while loan to government has the lowest volatility.

Figure 8 and 9 illustrate the Impulse response function in response to technology and government expenditure shock. Most of the selected variables as shown in the figure namely: deposit rate; loan rate on government; loan rate on patient households; loan rate on impatient households; loan rate on firm; consumption of patient households; consumption of impatient households; total consumption; total loan; loan to impatient households; loan to government; loan to firm; output; labor patient households; labor impatient households; total labor; investment; deposits; capital; rate of capital and wage rate.

Table 6: Mean and Standard Deviation of variables: Ramsey vs private market

| Variable | Ramsey Solution Mean | Ramsey Solution Std.Dev | Private Sector Solution Mean | Private Sector Solution Std.Dev |
|---|---|---|---|---|
| $C_p$ | 0.5090 | 0.0034 | 0.5059 | 0.0003 |
| $N_p$ | 0.3011 | 0.0045 | 0.3064 | 0.0029 |
| $C_i$ | 0.4672 | 0.0055 | 0.4696 | 0.0034 |
| $N_i$ | 0.3847 | 0.0052 | 0.3529 | 0.0038 |
| $R$ | 0.0351 | 0.0003 | 0.0351 | 0.0001 |
| $I$ | 0.3675 | 0.0093 | 0.3673 | 0.0029 |
| $y$ | 1.7200 | 0.0148 | 1.7191 | 0.0043 |
| $w$ | 1.7557 | 0.0145 | 1.7557 | 0.0066 |
| $r_p$ | 0.0101 | 0.0002 | 0.0101 | 0.0001 |
| $r_i$ | 0.0417 | 0.0006 | 0.0417 | 0.0005 |
| $r_f$ | 0.0417 | 0.0006 | 0.0417 | 0.0005 |
| $r_g$ | 0.0184 | 0.0002 | 0.0184 | 0.0001 |
| $L_f$ | 2.0833 | 0.0268 | 2.0833 | 0.0346 |
| $L_g$ | 2.7240 | 0.0061 | 2.7240 | 0.0007 |
| $L_i$ | 2.0833 | 0.0268 | 2.0833 | 0.0346 |
| $t$ | 0.3500 | 0.0000 | 0.3500 | 0.0000 |
| $k$ | 14.7001 | 0.0940 | 14.6927 | 0.0098 |
| $G$ | 0.3000 | 0.0000 | 0.3000 | 0.0000 |
| $A$ | 1.0000 | 0.0066 | 1.0000 | 0.0034 |
| $c$ | 0.9762 | 0.0080 | 0.9755 | 0.0036 |
| $N$ | 0.6858 | 0.0031 | 0.6854 | 0.0009 |
| $D$ | 6.8906 | 0.0506 | 6.8907 | 0.0692 |
| $\frac{L_i}{L}$ | 0.3023 | 0.0017 | 0.3023 | 0.0020 |
| $\frac{L_f}{L}$ | 0.3023 | 0.0017 | 0.3023 | 0.0020 |
| $\frac{L_g}{L}$ | 0.3953 | 0.0034 | 0.3953 | 0.0040 |
| $t_p$ | 0.2377 | 0.0146 | 0.2500 | 0.0000 |
| $t_i$ | 0.1123 | 0.0146 | 0.1000 | 0.0000 |

Figure 8 shows the impulse response of above variables to a positive technology shock. The red line indicates the technology shocks on optimal policy problem while the blue line shows the technology shocks on private market solution problem. Christiano et al. (2005) and Smets and Wouters (2007) showed that with positive technology shock output, consumption and investment will grow. However, interest rate is inversely related and it will decrease. As shown in this paper, not only consumption, output and investment but also wage rate has a positive effect. After the technology shocks, total consumption increases and then converges to steady state value because of the increase in consumption of both household- patient and impatient- due to wage income increase. In addition, loan to impatient household decreases and it may be because of the decrease of interest rate of impatient household and then decrease the repayment of interest rate with principle.

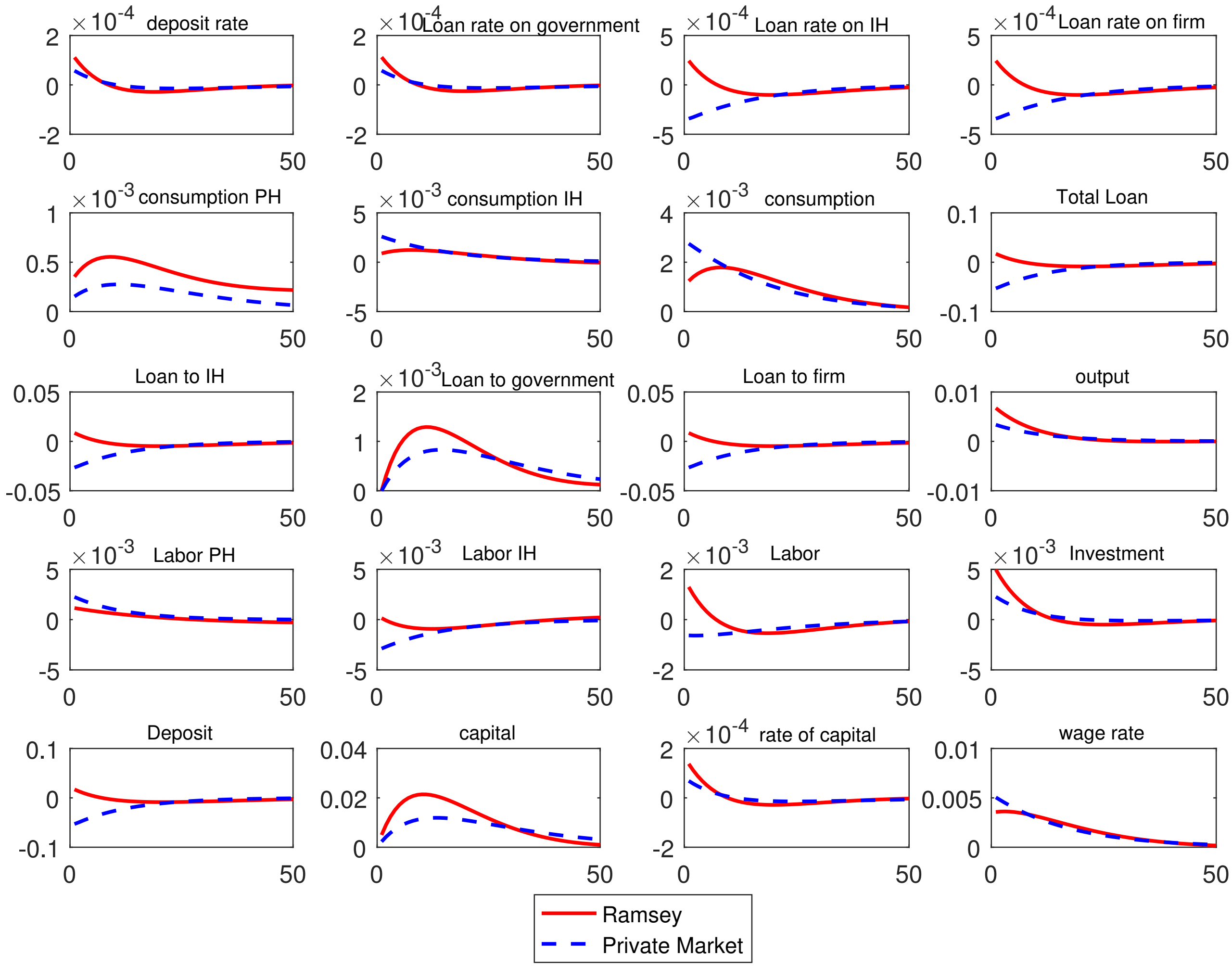


Figure 8: Impulse response to productivity shock: Ramsey vs private market

Figure 9 shows the impulse response of same variables to a positive government expenditure shocks. Glomm et al. (1997) focused on two different types of government expenditure: government expenditure enters as inputs in to production function (infrastructure); and, government expenditure enters as input in investment technology (education). However, in this paper, it is not differentiated any specific type of government expenditure.

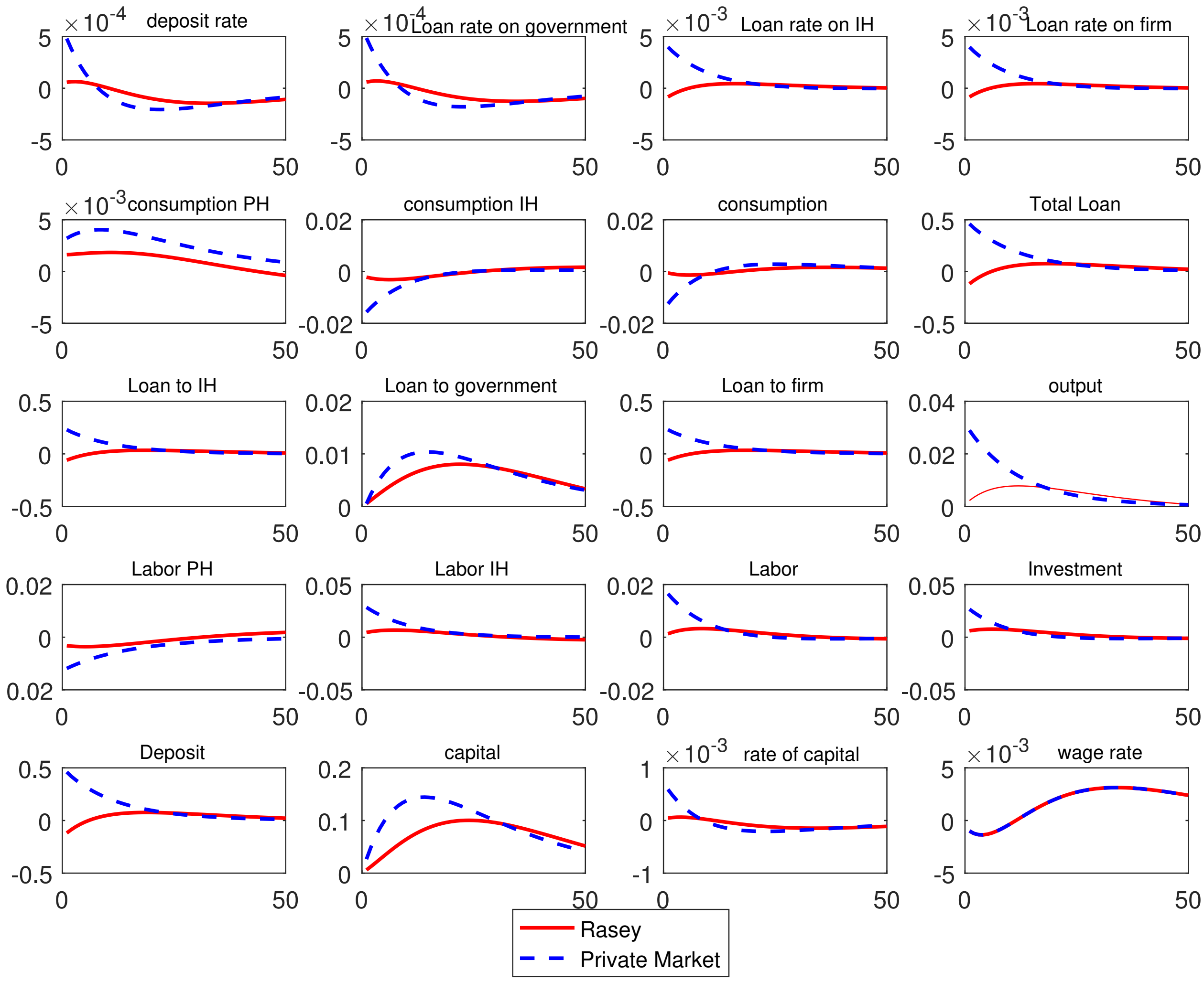


Figure 9: Impulse response to government expenditure shock: Ramsey vs private market

The red line of the figure, indicates the government expenditure shocks on optimal policy problem solution, and the blue line indicates the government expenditure shocks on private market solution problem. In this case, total consumption decrease after the positive technology shocks. Because, consumption of impatient household decreases. It may be due to the decrease of wage rate and increase the interest rate of impatient household.

# 7 Summary and Conclusion

The main intention of the paper is to characterize optimal credit allocation by commercial banks to impatient households, firm, and government. Calibration of the model parameters correspond to the European Union (EU) and the USA values and are taken from the several research papers. Discount factors of impatient households and firms were set equal. Both impatient households and firms behave as an identical agent with the assumption of equal risk factors.

As per the results, there is no difference of steady state values of loans to each agent and deposits under the optimal solution problem and private market solution. However, standard deviations are different. When considering the loan volatility of each agent, loans to impatient households have higher volatility than loans to government. The steady state of consumption, labor, investment and output are different under optimal policy and private market solutions. Consumption of patient households is higher than that of impatient households. However, labor of impatient households is higher than labor of patient households. The standard deviations of all variables, are higher under optimal policy problem than that of private market solutions. Further steady-state values of wage rate and output are similar in both cases.

The study revealed evidence for an important dynamic of the loan allocating process which entails the understanding of the risk factor behavior of commercial banks. The evidence shows, when risk is increasing for the impatient household or the firm the total allocation of loans of impatient house hold converges to zero, assuming the risk factor for the other two remains unchanged. However, the decrease of the risk of the government allocation of loan reached to its upper bound about 55% of the total loan allocation.

In a situation where the risk for all three agents are hypothetically equal and high (developing countries or poor countries), banks will allocate most of the overall loan(about 80%) to the government. However, in the situation, of equally low (developed countries or rich countries) banks allocate most of loan (75%) to households and firms. Further, interest rate (discount factor) plays a dominant role when there are low risk agents in the economy as when discount factor changes by small amount, percentage of loans will change in considerable amounts.

After considering the facts and figures carefully and applying them in to the model, which is suggested by this study, it is inevitable to state that the optimal credit allocation is mainly dependent on two factors: discount factor and the risk factor of the corresponding agents. The discount factor, to that end, does not plays a significant role in the process of loan allocation in the high-risk situation, yet, in the low risk situations, it has a considerable amount of effect on the loan allocation process. Therefore, this study provides a significant yet timely intervention to the current discourse of finding possible strategic solutions to the asset distribution of commercial banks to yield higher societal satisfactions.

# 8 Reference


1. Carrera, C.,and Vega, H., 2012. Inter-bank market and macroprudential tools in a dsge model. *Working Papers 2012-014*, Banco Central de Reserva del PerÂťu, June 2012.

2. Chari, V. V., Christiano, L.J., and Kehoe, P.J., 1991. Optimal fiscal and monetary policy some recent results. *Journal of Money, Credit and Banking*, 23, 519-39.

3. Christiano, L.J., Eichenbaum, M., and Charles, L. E., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 1131, 1-45.

4. Dib, A., 2010. Banks, credit market frictions, and business cycles. *Working Papers Bank of Canada*, 10-24.

5. Faia, E., 2007. Ramsey monetary policy with labor market frictions. *European Central Bank Working Paper Series 707*.

6. Falagiarda, M.,and Saia, A., 2013. Credit, Endogenous collateral and risky assets: a DSGE model. *Quaderni-Working Paper DSE No 916*, ISSN 2282-6483.

7. GalÃŋ, J., 1999. Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 249-271.

8. Gerali, A., Neri S, Sessa, L., and Signoretti F.M.,2010. Credit and banking in a DSGE model of the euro area. *Journal of Money, Credit and Banking*, 42, 107-141.

9. Gertler, M.,and Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics*, 3, 547-599.

10. Glomm,G., and Ravikumar, B., 1997. Productive government expenditures and long-run growth. *Journal of Economic Dynamics and Control*, 21, 183-204.

11. Goodhart, C., and Segoviano, M.A.,2009. Money and capital markets department. *IMF working paper*.

12. Iacoviello, M., 2005. House prices, borrowing constraints and monetary policy in the business cycle. *American Economic Review*, 95, 3, 739-64.

13. Iacoviello, M., 2015. Financial Business Cycles. *Review of Economic Dynamics*, 181, 140âĂŞ163.

14. Khan, A., King R., and Wolman. A. L., 2003. Optimal monetary policy. *Review of Economic Studies*, 60-4.

15. Kollintzas,T., Konstantakopoulou,I., and Tsionas, E., 2011. Stylized facts of money and credit over the business cycles. *Alied Financial Economics*, 21, 1735-1755.

16. Monacelli, T., 2006. Optimal monetary policy with collateralized household debt and borrowing constraints. *NBER working paper series*, 12470.

17. Schmitt-Grohe,S., and Uribe, M., 2004. Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114, 198-230.

18. Siu, H., 2004. Optimal fiscal and monetary policy with sticky prices. *Journal of Monetary Economics*, 51,3, 575-607.

19. Smets, F., and Wouters, R.,2007. Shocks and frictions in us business cycles: A Bayesian DSGE approach. *American Economic Review*, 973, 586-606.

# Appendix A: Solving Patient Household problem

Maximize lifetime utility of patient households:

$$E_0 \sum_{t=0}^{\infty} (\beta_p)^t U^p \left[C_{p,t}, N_{p,t}\right]$$

subject to the sequence of budget constraints of the form:

$$C_{p,t} + D_{t+1} + I_t = W_t N_{p,t} + R_t K_t + \left(1 + r_{p,t-1}\right) D_t + \Pi_t - T_{p,t}$$

and capital accumulation condition:

$$K_{t+1} = I_t + (1-\delta) K_t.$$

Then write the problem using Lagrangian multiplier as:

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \Bigg\{ & \beta_p^t \left(\ln C_{p,t} - \theta_p \frac{N_{p,t}^{1+\chi}}{1+\chi}\right) \\ & + \lambda_t \beta_p^t \left(W_t N_{p,t} + R_t K_t + \Pi_t - T_{p,t} + r_{p,t-1} D_t - C_{p,t} - K_{t+1} + (1-\delta) K_t - D_{t+1} + D_t\right) \Bigg\}.\end{aligned} \tag{A.1}$$

First order conditions w.r.t $\left\{C_{p,t}, N_{p,t}, D_{t+1}, K_{t+1}\right\}$

$$[CP_t] \quad \frac{\beta_p^t}{C_{p,t}} + \lambda_t \beta_p^t (-1) = 0 \Rightarrow \frac{1}{C_{p,t}} = \lambda_t \tag{A.2}$$

$$[NP_t] \quad \beta_p^t (-\theta_p) \frac{N_{p,t}^{\chi} (1+\chi)}{(1+\chi)} + \lambda_t \beta_p^t (W_t) = 0 \Rightarrow \theta_p N_{p,t}^{\chi} = \lambda_t W_t \tag{A.3}$$

$$[D_{t+1}] \quad \lambda_t \beta_p^t (-1) + E_t \lambda_{t+1} \beta_p^{t+1} \left(1 + r_{p,t}\right) \Rightarrow \lambda_t = E_t \lambda_{t+1} \beta_p \left(1 + r_{p,t}\right) \tag{A.4}$$

$$[K_{t+1}] \quad \lambda_t \beta_p^t (-1) + E_t \lambda_{t+1} \beta_p^{t+1} (1-\delta) + E_t \lambda_{t+1} \beta_p^{t+1} R_{t+1} = 0 \lambda_t = E_t \lambda_{t+1} \beta_p (1 - \delta + R_{t+1}) \tag{A.5}$$

Using (A2) and (A3),

$$\theta_p N_{p,t}^{\chi} C_{p,t} = W_t \theta_p N_{p,t}^{\chi} = \frac{W_t}{C_{p,t}} \tag{A.6}$$

Using (A2) and (A5),

$$\frac{1}{C_{p,t}} = E_t \frac{1}{C_{p,t+1}} \beta_p \left(R_{t+1} + 1 - \delta\right) \tag{A.7}$$

Using (A2) and (A5),

$$\frac{1}{C_{p,t}} = E_t \frac{1}{C_{p,t+1}} \beta_p \left(1 + r_{p,t}\right) \tag{A.8}$$

# Appendix B: Solving Impatient Household Problem

Maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} {\beta_I}^t U^I \left[C_{i,t}, N_{i,t}\right]$$

subject to the sequence of budget constraints of the form:

$$C_{i,t} + \left(1 + r_{i,t-1}\right) L_{i,t} = W_t N_{i,t} + L_{i,t+1} + T_{i,t}.$$

Then, write the problem using Lagrangian multiplier as:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \left\{ \beta_I^t \left( \ln C_{i,t} - \theta_I \frac{N_{i,t}^{1+\chi}}{1+\chi} \right) + \lambda_t \beta_p^t \left( W_t N_{i,t} - T_{2t} - r_{i,t-1} L_{i,t} - C_{i,t} + L_{i,t+1} - L_{i,t} \right) \right\} \tag{B.1}$$

Then the first order condition w.r.t $\left\{C_{i,t}, N_{i,t}, L_{i,t+1}\right\}$ .

$$[CI_t] \quad \frac{\beta_I^t}{C_{i,t}} + \lambda_t . \beta_I^t (-1) = 0 \Rightarrow \frac{1}{C_{i,t}} = \lambda_t \tag{B.2}$$

$$[NI_t] \quad \beta_I^t (-\theta_p) \frac{N_{i,t}^{\chi} (1+\chi)}{(1+\chi)} + \lambda_t \beta_I^t (W_t) = 0 \Rightarrow \theta_I N_{i,t}^{\chi} = \lambda_t W_t \tag{B.3}$$

$$[L_{i,t+1}] \quad \lambda_t \beta_I^t (-1) - E_t \lambda_{t+1} \beta_I^{t+1} \left(1 + r_{i,t}\right) = 0 \Rightarrow \lambda_t = E_t \lambda_{t+1} \beta_I \left(1 + r_{i,t}\right) \tag{B.4}$$

By (B2) and (B3),

$$\theta_I N_{i,t}^{\chi} = \frac{W_t}{C_{i,t}} \tag{B.5}$$

By (B2) and (B4),

$$\frac{1}{C_{i,t}} = E_t \frac{1}{C_{i,t+1}} \beta_I \left(1 + r_{i,t}\right) \tag{B.6}$$

# Appendix C: Solving Firms Problem

Firm problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} M_t \left\{ A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - R_t K_t + L_{f,t+1} - \left(1 + r_{f,t-1}\right).L_{f,t} \right\} \tag{C.1}$$

Then first order conditions w.r.t $\left\{N_t, K_t, L_{f,t+1}\right\}$

$$[N_t] \quad (1-\alpha) A_t N_t^{-\alpha} K_t^{\alpha} - W_t = 0 \Rightarrow (1-\alpha) N_t^{-\alpha} A_t K_t^{\alpha} = W_t \tag{C.2}$$

$$[K_t] \quad \alpha A_t . N_t^{1-\alpha} K_t^{\alpha-1} - R_t = 0 \Rightarrow \alpha . N_t^{1-\alpha} A_t K_t^{\alpha-1} = R_t \tag{C.3}$$

$$\left[L_{f,t+1}\right] \quad M_t - E_t M_{t+1} \left(1 + r_{f,t}\right) = 0 \Rightarrow \frac{1}{C_{i,t}} = \beta_f E_t \frac{1}{C_{i,t+1}} \left(1 + r_{f,t}\right) \tag{C.4}$$

# Appendix D: Bank Problem

Bank problem is to maximize profit :

$$\mathcal{L} = \sum_{t=0}^{\infty} B_t \Bigg( D_{t+1} + \left(1 + r_{f,t-1}\right) L_{f,t} + \left(1 + r_{g,t-1}\right) L_{g,t} + \left(1 + r_{i,t-1}\right) L_{i,t} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} - \left(1 + r_{p,t-1}\right) D_t - \frac{\phi_f}{2} L_{f,t+1}^2 - \frac{\phi_g}{2} L_{g,t+1}^2 - \frac{\phi_i}{2} L_{i,t+1}^2 \Bigg)$$

subject to the balance sheet identity

$$D_{t+1} = L_{f,t+1} + L_{g,t+1} + L_{i,t+1}$$

Then rewrite the problem again with Lagrange multiplier as:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} B_t \Bigg( D_{t+1} + \left(1 + r_{f,t-1}\right) L_{f,t} + \left(1 + r_{g,t-1}\right) L_{g,t} + \left(1 + r_{i,t-1}\right) L_{i,t} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} - \left(1 + r_{p,t-1}\right) D_t - \frac{\phi_f}{2} L_{f,t+1}^2 - \frac{\phi_g}{2} L_{g,t+1}^2 - \frac{\phi_i}{2} L_{i,t+1}^2 + \lambda_t \left( D_{t+1} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} \right) \Bigg) \tag{D.1}$$

Then first order conditions w.r.t $\left\{D_{t+1}, L_{f,t+1}, L_{g,t+1}, L_{i,t+1}\right\}$ are:

$$[D_{t+1}] \quad B_t (1 + \lambda_t) - B_{t+1} \left(1 + r_{p,t}\right) = 0 \Rightarrow B_t (1 + \lambda_t) = B_{t+1} \left(1 + r_{p,t}\right) \tag{D.2}$$

$$\left[L_{f,t+1}\right] \quad - B_t \frac{2}{2} \phi_f L_{f,t+1} - \lambda_t B_t + \left(1 + r_{f,t}\right) B_{t+1} - B_t = 0$$

substitute $\lambda_t = \beta_B \left(1 + r_{p,t}\right) - 1$ to the above equation:

$$B_t \phi_f L_{f,t+1} = B_{t+1} \left( r_{f,t} - r_{p,t} \right) \tag{D.3}$$

Similarly, we can find the other two first order condition as follows:

$$[L_{g,t+1}] \quad B_t \phi_g L_{g,t+1} = B_{t+1} \left( r_{g,t} - r_{p,t} \right) \tag{D.4}$$

$$[L_{i,t+1}] \quad B_t \phi_i L_{i,t+1} = B_{t+1} \left( r_{i,t} - r_{p,t} \right) \tag{D.5}$$

Then by substituting $B_t$ and $B_{t+1}$ we can get the following three equations

$$\phi_f \frac{1}{C_{p,t}} L_{f,t+1} = \beta_B \frac{1}{C_{p,t+1}} \left( r_{f,t} - r_{p,t} \right) \tag{D.6}$$

$$\phi_g \frac{1}{C_{p,t}} L_{g,t+1} = \beta_B \frac{1}{C_{p,t+1}} \left( r_{g,t} - r_{p,t} \right) \tag{D.7}$$

$$\phi_i \frac{1}{C_{p,t}} L_{i,t+1} = \beta_B \frac{1}{C_{p,t+1}} \left( r_{i,t} - r_{p,t} \right) \tag{D.8}$$

# Appendix E: Derive the Resource Constraint

First consider the government budget constraint:

$$G_t + r_{g,t-1} L_{g,t} = \left( T_{p,t} + T_{i,t} \right) + L_{g,t+1} - L_{g,t} \tag{E.1}$$

Then substitute $T_{p,t}$ and $T_{i,t}$ from both households budget constraints

$$\begin{aligned} G_t + r_{g,t-1} L_{g,t} = & \left( W_t N_{p,t} + R_t K_t + \left(1 + r_{p,t-1}\right) D_t + \pi_t^b - C_{p,t} - D_{t+1} - I_t \right) \\ & + \left( W_t N_{i,t} + L_{i,t+1} + \pi_t^f - C_{i,t} - \left(1 + r_{i,t-1}\right) L_{i,t} \right) + L_{g,t+1} - L_{g,t} \end{aligned} \tag{E.2}$$

Next substitute following two profit functions of firms and bank to the equation $E.1$

$$\begin{aligned} \pi_t^b = & D_{t+1} + \left(1 + r_{f,t-1}\right) L_{f,t} + \left(1 + r_{g,t-1}\right) L_{g,t} + \left(1 + r_{i,t-1}\right) L_{i,t} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} \\ & - \left(1 + r_{p,t-1}\right) D_t - \frac{\phi_f}{2} L_{f,t+1}^2 - \frac{\phi_g}{2} L_{g,t+1}^2 - \frac{\phi_i}{2} L_{i,t+1}^2 + \lambda_t \left( D_{t+1} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} \right) \end{aligned} \tag{E.3}$$

$$\pi_t^f = A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - R_t K_t + L_{f,t+1} - \left(1 + r_{f,t-1}\right) . L_{f,t} \tag{E.4}$$

Then after simplify, we can get the following resource constraint

$$Y_t = C_t + I_t + G_t + \frac{\phi_f}{2} L_{f,t}^2 + \frac{\phi_g}{2} L_{g,t}^2 + \frac{\phi_i}{2} L_{i,t}^2 \tag{E.5}$$

# Appendix F: Solving Ramsey Planer Problem

Define welfare function as:

$$W = \omega \sum_{t=0}^{\infty} \beta_p^t \left[ \ln C_{p,t} - \theta_p \frac{N_{p,t}^{1+\chi}}{1+\chi} \right] + (1-\omega) \sum_{t=0}^{\infty} \beta_I^t \left[ \ln C_{i,t} - \theta_I \frac{N_{i,t}^{1+\chi}}{1+\chi} \right] \tag{F.6}$$

Then maximize social welfare function subject to first order condition of the private sector and resource constraints.

$$\begin{aligned}
&Max\ E_t \sum_{t=0}^{\infty} \left( \omega \beta_p^t \left[ \ln C_{p,t} - \theta_p \frac{N_{p,t}^{1+\chi}}{1+\chi} \right] + (1-\omega)\, \beta_I^t \left[ \ln C_{i,t} - \theta_I \frac{N_{i,t}^{1+\chi}}{1+\chi} \right] \right) \\
&+ \Delta^t \lambda_{1,t} \left( \theta_p N_{p,t}^{\chi} - \frac{W_t}{C_{p,t}} \right) \\
&+ \Delta^t \lambda_{2,t} \left( \frac{1}{C_{p,t}} \right) - \Delta^{t-1} \lambda_{2,t-1} \left( \frac{\beta_p (1-\delta+R_t)}{C_{p,t}} \right) \\
&+ \Delta^t \lambda_{3,t} \left( \frac{1}{C_{p,t}\left(1+r_{p,t}\right)} \right) - \Delta^{t-1} \lambda_{3,t-1} \left( \frac{\beta_p}{C_{p,t}} \right) \\
&+ \Delta^t \lambda_{4,t} \left( \theta_I N_{i,t}^{\chi} - \frac{W_t}{C_{i,t}} \right) \\
&+ \Delta^t \lambda_{5,t} \left( \frac{1}{C_{i,t}\left(1+rI_t\right)} \right) + \Delta^{t-1} \lambda_{5,t-1} \left( \frac{\beta_I}{C_{i,t}} \right) \\
&+ \Delta^t \lambda_{6,t} \left( (1-\alpha) A_t K_t^{\alpha} \left(N_{p,t}+N_{i,t}\right)^{-\alpha} - W_t \right) + \Delta^{t+1} \lambda_{7,t+1} \Big( (1-\alpha) A_t K_{t+1}^{\alpha} \left(N_{p,t+1}+N_{i,t+1}\right)^{-\alpha} \Big) \\
&+ \Delta^t \lambda_{7,t} \left( 1 - \beta_f \left(1+r_{f,t}\right) \right) \\
&+ \Delta^t \lambda_{8,t} \left( \alpha.A_t (N_{i,t}^{1-\alpha} + N_{p,t}^{1-\alpha}) K_t^{\alpha-1} - R_t \right) + \Delta^{t+1} \lambda_{8,t+1} \left( \alpha.A_{t+1} (N_{p,t+1}+N_{i,t+1})^{1-\alpha} K_{t+1}^{\alpha-1} \right) \\
&+ \Delta^t \lambda_{9,t} \left( G_t - T_t + r_{g,t-1} L_{g,t} - L_{g,t+1} - L_{g,t} \right) + \Delta^{t+1} \lambda_{9,t+1} \left( r_{g,t} L_{g,t+1} - L_{g,t+1} \right) \\
&+ \Delta^t \lambda_{10,t} \big( \phi_f \frac{1}{C_{p,t}} L_{f,t+1} - \beta_B \frac{1}{C_{p,t+1}} \left( r_{f,t} - r_{p,t} \right) \big) \\
&+ \Delta^t \lambda_{11,t} \big( \phi_i \frac{1}{C_{p,t}} L_{i,t+1} - \beta_B \frac{1}{C_{p,t+1}} \left( r_{i,t} - r_{p,t} \right) \big) \\
&+ \Delta^t \lambda_{12,t} \big( \phi_g \frac{1}{C_{p,t}} L_{g,t+1} - \beta_B \frac{1}{C_{p,t+1}} \left( r_{g,t} - r_{p,t} \right) \big) \\
&+ \Delta^t \lambda_{13,t} \left( A_t K_t^{\alpha} (N_{p,t}+N_{i,t})^{1-\alpha} - \left(C_{i,t}+C_{p,t}\right) - G_t - K_{t+1} + (1-\delta) K_t \right) \\
&+ \Delta^t \lambda_{13,t+1} \left( A_{t+1} K_{t+1}^{\alpha} (N_{p,t+1}+N_{i,t+1})^{1-\alpha} + (1-\delta) K_{t+1} \right)
\end{aligned} \tag{F.7}$$

Then find the first order condition with respect to $\{C_{p,t}, C_{i,t}, N_{p,t}, N_{i,t}, K_{t+1}, L_{f,t}, L_{i,t}, L_{g,t}, w_t, R_t, r_{p,t}, r_{i,t}, r_{g,t}, r_{f,t}\}$