Optimal Credit Allocations

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How commercial banks should allocate their deposits among three different agents: households, firms, and the government to achieve socially optimal allocation?

- Optimal allocation depends on the discount factor and risk factor
- 60% of total loan to impatient households and firms, rest government

My Contribution

Identify socially optimal loan allocations to agents in the economy by the commercial banks

Outline

- Model
- Results
- Conclusion

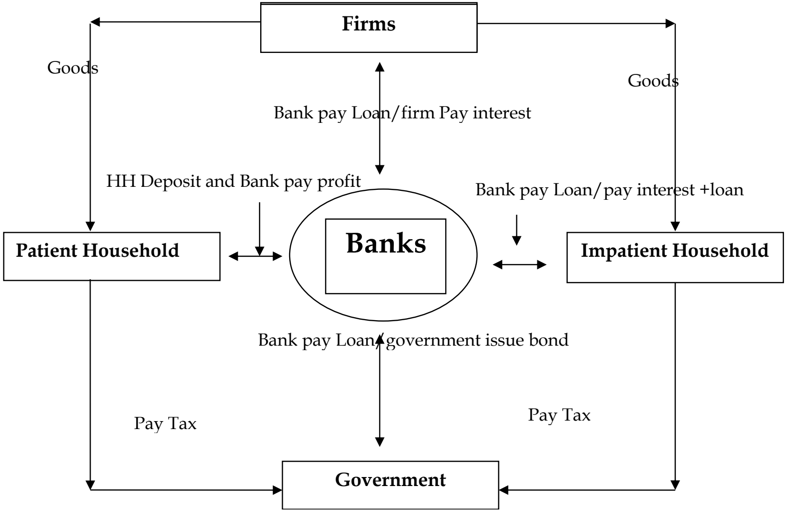

Figure 1: Graphical view of the overall model

- Households maximize their expected discounted lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} (\beta_p)^t \left[\ln C_{p,t} - \theta_p \frac{N_{p,t}^{1+\chi}}{1+\chi} \right]$$

$$\text{s.t. } C_{p,t} + D_{t+1} + I_t = W_t N_{p,t} + R_t K_t + \left(1 + r_{p,t-1}\right) D_t + \Pi_t - T_{p,t}$$

$$\text{and } K_{t+1} = I_t + (1-\delta) K_t$$

First order conditions give the following equilibrium conditions:

$$\theta_p N_{p,t}^{\chi} = \frac{W_t}{C_{p,t}} \quad (1)$$

$$\frac{1}{C_{p,t}} = \beta_p E_t \left[\frac{1}{C_{p,t+1}} (R_{t+1} + 1 - \delta) \right] \quad (2)$$

$$\frac{1}{C_{p,t}} = \beta_p E_t \left[\frac{1}{C_{p,t+1}} \left(1 + r_{p,t}\right) \right] \quad (3)$$

- Households maximize their expected discounted lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} (\beta_I)^t \left[\ln C_{i,t} - \theta_I \frac{N_{i,t}^{1+\chi}}{1+\chi} \right]$$

$$\text{s.t. } C_{i,t} + \left(1 + r_{i,t-1}\right) L_{i,t} = W_t N_{i,t} + L_{i,t+1} - T_{i,t}$$

First order conditions give the following equilibrium conditions:

$$\theta_I N_{i,t}^{\chi} = \frac{W_t}{C_{i,t}} \tag{4}$$

$$\frac{1}{C_{i,t}} = \beta_I E_t \left[\frac{1}{C_{i,t+1}} \left(1 + r_{i,t}\right) \right] \tag{5}$$

- Firms' problem can be expressed by the following maximization problem:

$$E_0 \sum_{t=0}^{\infty} M_t \left(A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - R_t K_t + L_{f,t+1} - \left(1 + r_{f,t-1}\right) L_{f,t} \right)$$

Define the stochastic discount factor as:

$$M_t = \beta_f^t \frac{U^{I'}\left(C_{i,t}\right)}{U^{I'}\left(C_{i,0}\right)}$$

First order conditions give the factor prices equal to their marginal products:

$$(1-\alpha) N_t^{-\alpha} A_t K_t^{\alpha} = W_t \tag{6}$$

$$\alpha N_t^{1-\alpha} A_t K_t^{\alpha-1} = R_t \tag{7}$$

$$\frac{1}{C_{i,t}} = \beta_f E_t \left[\frac{1}{C_{i,t+1}} \left(1 + r_{f,t}\right) \right] \tag{8}$$

- Government's budget constraint can be written as:

$$G_t + r_{g,t-1} L_{g,t} = T_t + L_{g,t+1} - L_{g,t} \tag{9}$$

- Banks maximize the expected discounted profit $E_0 \sum_{t=0}^{\infty} B_t \pi_t$.
- Hence, bank's problem can be written as:

$$
\begin{aligned}
E_0 \sum_{t=0}^{\infty} B_t \Big(& D_{t+1} + (1 + r_{f,t-1}) L_{f,t} + (1 + r_{g,t-1}) L_{g,t} \\
& + (1 + r_{i,t-1}) L_{i,t} - L_{f,t+1} - L_{g,t+1} - L_{i,t+1} \\
& - (1 + r_{p,t-1}) D_t - \frac{\phi_f}{2} L_{f,t+1}^2 - \frac{\phi_g}{2} L_{g,t+1}^2 - \frac{\phi_i}{2} L_{i,t+1}^2 \Big) \\
& \text{s.t. } D_{t+1} = L_{f,t+1} + L_{g,t+1} + L_{i,t+1}
\end{aligned}
$$

Define the stochastic discount factor as: $B_t = \beta_B^t \frac{U^{P'}(C_{p,t})}{U^{P'}(C_{p,0})}$

First order conditions give the following equilibrium:

$$\phi_f \frac{1}{C_{p,t}} L_{f,t+1} = \beta_B \frac{1}{C_{p,t+1}} \left(r_{f,t} - r_{p,t} \right) \tag{10}$$

$$\phi_g \frac{1}{C_{p,t}} L_{g,t+1} = \beta_B \frac{1}{C_{p,t+1}} \left(r_{g,t} - r_{p,t} \right) \tag{11}$$

$$\phi_i \frac{1}{C_{p,t}} L_{i,t+1} = \beta_B \frac{1}{C_{p,t+1}} \left(r_{i,t} - r_{p,t} \right) \tag{12}$$

1. Productivity

$$\ln A_t = (1 - \rho_a) \ln A + \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{13}$$

where $A = 1$, $0 < \rho_a < 1$ is the AR(1) persistence parameter and $\varepsilon_{a,t} \sim N\left(0, \sigma_a^2\right)$.

2. Government expenditure

$$\ln g_t = \left(1 - \rho_g\right) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \tag{14}$$

where $g = 1$, $0 < \rho_g < 1$, is the AR(1) persistence parameter and $\varepsilon_{g,t} \sim N\left(0, \sigma_g^2\right)$.

- Ramsey planer's maximization problem can be written as:

$$W = \omega \sum_{t=0}^{\infty} \beta_p^t U^p \left(C_{p,t}, N_{p,t}\right) + (1-\omega) \sum_{t=0}^{\infty} \beta_I^t U^I \left(C_{i,t}, N_{i,t}\right)$$

subject to the equilibrium conditions (1) - (12) and resource constraint for a given stochastic process $\{A_t, G_t\}_{t=0}^{\infty}$.

Table 1: Calibrated parameters for the model

Parameters	Value	Description
β_p	0.99	Subjective discount factor for the patient household
β_I	0.96	Subjective discount factor for the impatient household
β_f	0.96	Subjective discount factor for firms
β_B	0.99	Subjective discount factor for banks
α	0.30	Capital share of production
χ	0.35	Elasticity of labor supply with respect to wage
θ_p	5.25	Disutility of labor by patient household
θ_I	5.25	Disutility of labor by impatient household
ϕ_I	0.015	Risk factor of impatient household on loan
ϕ_g	0.003	Risk factor of government on loan
ϕ_f	0.015	Risk factor of firm on loan
ω	0.5	Ramsey preference weight
δ	0.025	Depreciation of capital
ρ_a	0.92	Serial correlation of technology shocks
ρ_g	0.92	Serial correlation of government expenditure shocks
y_{ss}	1.5	Steady state of output
σ_z	0.0026	Standard deviation of the innovation to ln(z)
σ_u	0.0018	Standard deviation of the innovation to government expenditure

RESULTS: Comparison of Private Sector and Ramsey Planner Solution

Table 2: Mean and standard deviation of variables: Ramsey vs private market

	Ramsey Solution		Private Sector Solution	
Variable	Mean	Std.Dev	Mean	Std.Dev
C_p	0.5090	0.0034	0.5059	0.0003
N_p	0.3011	0.0045	0.3064	0.0029
C_i	0.4672	0.0055	0.4696	0.0034
N_i	0.3847	0.0052	0.3529	0.0038
R	0.0351	0.0003	0.0351	0.0001
I	0.3675	0.0093	0.3673	0.0029
y	1.7200	0.0148	1.7191	0.0043
w	1.7557	0.0145	1.7557	0.0066
r_p	0.0101	0.0002	0.0101	0.0001
r_i	0.0417	0.0006	0.0417	0.0005
r_f	0.0417	0.0006	0.0417	0.0005
r_g	0.0184	0.0002	0.0184	0.0001
L_f	2.0833	0.0268	2.0833	0.0346
L_g	2.7240	0.0061	2.7240	0.0007
L_i	2.0833	0.0268	2.0833	0.0346
t	0.3500	0.0000	0.3500	0.0000
k	14.7001	0.0940	14.6927	0.0098
G	0.3000	0.0000	0.3000	0.0000
A	1.0000	0.0066	1.0000	0.0034
c	0.9762	0.0080	0.9755	0.0036
N	0.6858	0.0031	0.6854	0.0009
D	6.8906	0.0506	6.8907	0.0692
$\frac{L_i}{L}$	0.3023	0.0017	0.3023	0.0020
$\frac{L_f}{L}$	0.3023	0.0017	0.3023	0.0020
$\frac{L_g}{L}$	0.3953	0.0034	0.3953	0.0040
t_p	0.2377	0.0146	0.2500	0.0000
t_i	0.1123	0.0146	0.1000	0.0000

Table 3: Comparisons of loan ratios of each agent: Ramsey vs market

	Ramsey Solution		Private Sector Solution	
	Mean	Std.Dev	Mean	Std.Dev
Loan to Impatient HH/Total Loan	30.23%	0.0017	30.23%	0.002
Loan to Firm/Total Loan	30.23%	0.0017	30.23%	0.002
Loan to Government/Total Loan	39.53%	0.0034	39.53%	0.004

Table 4: Comparison of loan ratio to toal loan according to different risk of Ramsey optimal policy problem

Equal risk among all agents	0.015	0.026	0.1	1	2
Loan to Impatient HH/Total Loan	36.68%	33.34%	24.41%	11.16%	8.40%
Loan to Firm/Total Loan	36.68%	33.34%	24.41%	11.16%	8.40%
Loan to Government/Total Loan	26.65%	33.31%	51.18%	77.68%	83.21%

RESULTS: Effect of Discount Factor on Risk

Table 5: Comparison of loan ratio to total loan according to discount factor in high risk region (ϕ_i=2, ϕ_g=0.4, ϕ_f=2) of Ramsey optimal policy problem

Impatient HH Discount Factor	Loan Impatient HH/Total Loan	Loan Firm/Total Loan	Loan Gov/Total Loan
0.9	16.55%	5.17%	78.28%
0.93	11.34%	5.49%	83.17%
0.95	7.70%	5.72%	86.58%
0.97	3.93%	5.95%	90.12%
0.989	1.00%	6.13%	92.87%

Table 6: Comparison of loan ratio to total loan according to discount factor in low risk region ($\phi_i = 0.015$, $\phi_g = 0.003$, $\phi_f = 0.015$) of Ramsey optimal policy problem

Impatient HH Discount Factor	Loan Impatient HH/Total Loan	Loan Firm/Total Loan	Loan Gov/Total Loan
0.9	58.10%	18.16%	23.74%
0.93	47.22%	22.87%	29.91%
0.95	36.86%	27.36%	35.77%
0.97	22.24%	33.70%	44.06%
0.989	6.58%	40.49%	52.94%

RESULTS: Effect of Risk on Loan Allocation

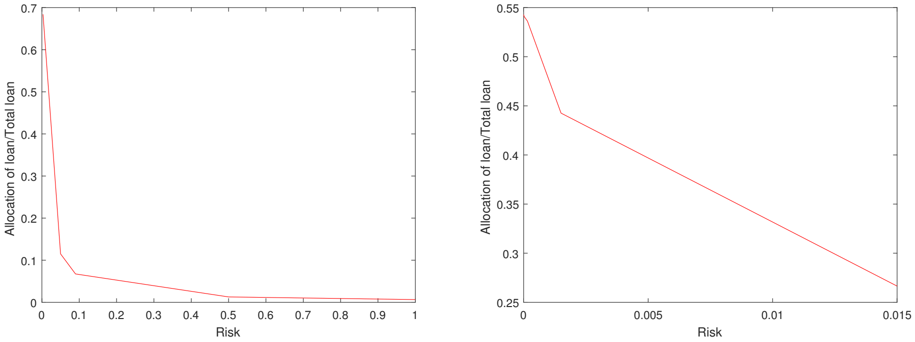

Figure 1: Loan allocation to impatient HH at different risk (left:$\phi_i = 0.003 - 1.00, \phi_g = 0.003, \phi_f = 0.015$) and Loan allocation to government at different risk (right: $\phi_i = 0.015, \phi_g = 0.015 - 0.000015, \phi_f = 0.015$)

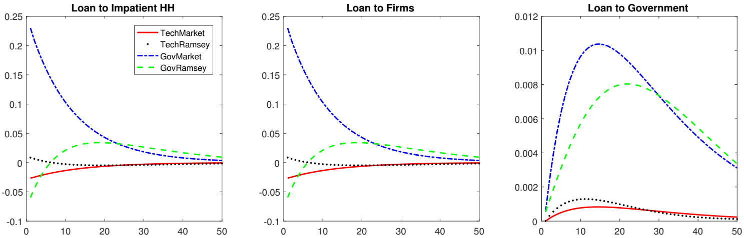

Figure 2: The impulse response functions to technology shocks and government expenditure shocks under Ramsey equilibrium and private market equilibrium

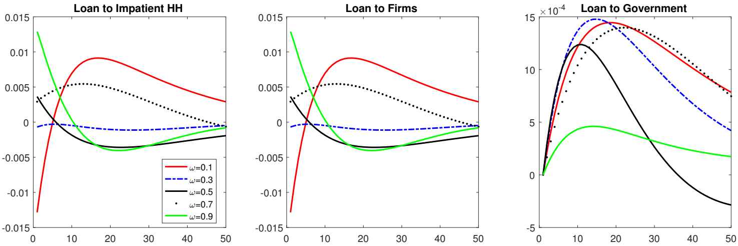

Figure 3: The impulse response functions to technology shocks for different weights

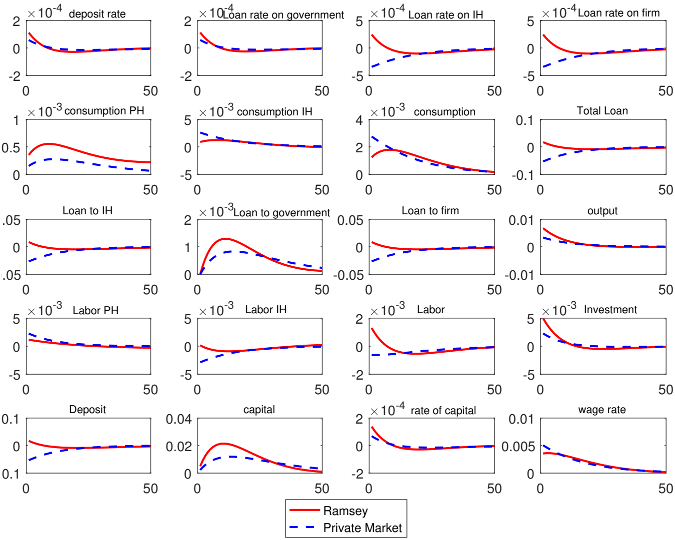

Figure 4: Impulse response to Tech-shock: Ramsey vs private market

RESULTS: Impulse Response Function to Gov-Shock

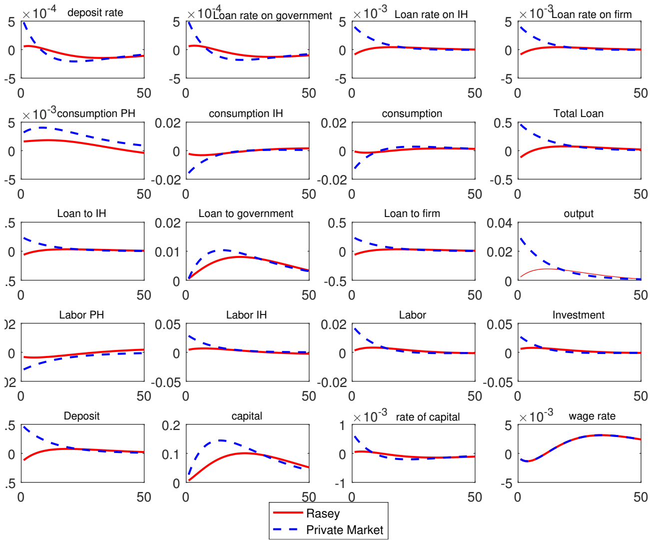

Figure 5: Impulse response to government expenditure shock: Ramsey vs private market

- Optimal credit allocation is mainly dependent on two factors
  - Discount factor
  - Risk factor
- Discount factor does not exert an important influence on optimal loan allocation when risk is high but, highly influential in the presence of low risk
- When the risk of households increases, optimal loan to households converges to zero.
- When the risk of the government decreases, the optimal loan to government reaches its upper bound of 55% of total loans
- 60% of the total loan should be allocated equally between households and firms and the rest should be allocated to the government

Thank you!